DIRECT DETERMINATION OF RANGE FROM CURRENT NUCLEAR OVERPRESSURE EQUATIONS

THESIS

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Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Operations Research

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March 1988

Approved for public release; distribution unlimited

Preface

Nuclear survivability of Air Force systems relies heavily on the quantification of those nuclear weapons effects that impact directly on the system's performance. Air shock, commonly referred to as peak overpressure, is one of the major nuclear weapons effects. Currently available equations calculate peak overpressure as a function of ground range from ground zero and weapon height of burst.

The purpose of this study was to develop a direct, non-iterative method for computing ground range as a function of peak overpressure and height of burst. The need for this inverse capability originated at the Air Force Center for Studies and Analysis, Missile Division.

Five polynomial equations were developed through curve fitting techniques to cover almost the entire range of the data. These five equations were combined into a computer program which computed the ground range directly. The program, written in Fortran 77 computer language, can be used separately or modified into a subroutine and incorporated within a larger program.

In performing the experimentation and writing of this thesis, I have had a great deal of help from others. I am especially grateful to my fellow classmates who provided a wealth of information about computers that I was lacking. I

am indebted to my thesis advisors, Lt Col R. F. Tuttle and Maj J. R. Litko for their assistance and sound advice. I wish to thank Maj W. D. Davis, Studies and Analysis, for his guidance and support. Finally, I wish to thank my wife, Jessica, who did all she could to support this endeavor.

Roger S. Wolczek

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Abstract

The Brode expression determines peak overpressure as a function of scaled ground range from ground zero and scaled height of burst for nuclear explosions occurring at low altitudes. To calculate the scaled ground range as a function of peak overpressure and scaled height of burst presently requires an iterative numerical method to invert the Brode expression. This study developed analytical expressions to directly compute scaled ground range from ground zero as a function of peak overpressure and scaled height of burst for a nuclear explosion.

Since the Brode expression was an empirical fit of actual and predicted data, a curve fitting approach was selected over attempting to mathematically invert the expression. The Brode expression was used to both generate the data and evaluate the quality of any new expression. Acceptable error was specified as ten percent of the actual ground range for those regions of interest. The data range was sufficiently large to warrant breaking the problem up into five smaller segments. Each segment of the problem was solved by using least squares curve fitting on the SAS System.

Five analytical expressions in the form of polynomial equations were developed spanning peak overpressures from 1

to 100,000 psi. These polynomial equations were then combined into a Fortran 77 computer program which generated ground range directly from inputs of weapon yield, peak overpressure, and weapon burst height.

In most cases the error of the new approximation was well below ten percent of the actual ground range. There were two instances where the error was 10.9 and 11.5 percent of the ground range. These two cases were isolated and not indicative of the overall fit.

Besides developing analytical expressions for computing ground range, a methodology for curve fitting a three dimensional surface was investigated. The methodology fits two independent variables simultaneously against a dependent variable rather than the more common one to one fit.

DIRECT DETERMINATION OF RANGE FROM

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I. Introduction

Issue of Concern

Survivability and vulnerability studies of weapons systems is an issue of concern throughout the United States Air Force. Consequently, much effort is devoted to the numerical computation of weapons effects. The Air Force Center for Studies and Analysis, Missile Division (AF/SASM), currently uses a series of complex mathematical expressions to calculate the probability of damage (Pd) from nuclear explosions. These mathematical expressions, designed for computer use, are the result of empirical data fits of available and estimated nuclear blast data. One such expression computes maximum pressure values of the shock front (peak overpressure) at the Earth's surface as a function of distance from ground zero and weapon burst height for a one kiloton explosion in a standard sea-level atmosphere. The probability of damage calculation requires an inverse capability of this expression to compute distance as a function of peak overpressure and burst height.

AF/SASM currently uses an iterative bisection computer routine to find the distance. However, the iteration slows the overall probability of damage calculation and produces some error dependent on specified accuracy limits of the computer routine. Therefore, personnel at AF/SASM desire a direct method to compute distance (or range) given values of peak overpressure and weapon burst height.

Specific Problem

There is a need to develop a direct, non-iterative method for computing scaled ground range from ground zero given the peak overpressure and scaled burst height.

Literature Review

The mathematical expression for peak overpressure that AF/SASM uses in its probability of damage calculations can be found on pages 60 through 71 of PSR Report 1419-3 (1). For the remainder of this thesis, this expression will be called the overpressure function. It takes the following form:

$$P = \frac{10.47}{r^{a(z)}} + \frac{b(z)}{r^{c(z)}} + \frac{d(z) \times e(z)}{1 + f(z) \times r^{g(z)}} + h(z,r,y) + \frac{j(y)}{r^{k(y)}} \quad (1)$$

where

P = overpressure in psi (pounds per square inch)

r = scaled slant range in kilofeet per cube-root kiloton and = $(x^2 + y^2)^{.5}$

x = scaled ground range in kilofeet per cube-root kiloton, or GR/m/1000

y = scaled burst height in kilofeet per cube-root kiloton, or H/m/1000

m = $W^{1/3}$ in cube-root kilotons (the scale factor)

W = yield in kilotons

GR = ground range in feet

H = burst height in feet

z = H/GR = y/x

and where

$$a(z) = 1.22 - \frac{3.908z^2}{1 + 810.2z^5}$$

$$b(z) = 2.321 + \frac{6.195z^{18}}{1 + 1.113z^{18}} - \frac{0.03831z^{17}}{1 + 0.02415z^{17}} + \frac{0.6692}{1 + 4164z^8}$$

$$c(z) = 4.153 - \frac{1.149z^{18}}{1 + 1.641z^{18}} - \frac{1.1}{1 + 2.771z^{2.5}}$$

$$d(z) = -4.166 + \frac{25.76z^{1.75}}{1 + 1.382z^{18}} + \frac{8.257z}{1 + 3.219z}$$

$$e(z) = 1 - \frac{0.004642z^{18}}{1 + 0.003886z^{18}}$$

$$f(z) = 0.6096 + \frac{2.879z^{9.25}}{1 + 2.359z^{14.5}} - \frac{17.15z^2}{1 + 71.66z^3}$$

$$g(z) = 1.83 + \frac{5.361z^2}{1 + 0.3139z^6}$$

$$h(z,r,y) = \frac{8.808z^{1.5}}{1 + 154.5z^{3.5}} - \frac{0.2905 + 64.67z^{5}}{1 + 441.5z^{5}} - \frac{1.389z}{1 + 49.03z^{5}}$$

$$+ \frac{1.094r^{2}}{(781.2 - 123.4r + 37.98r^{1.5} + r^{2})(1 + 2y)}$$

$$j(y) = \frac{0.000629y^{4}}{3.493x10^{-9} + y^{4}} - \frac{2.67y^{2}}{1 + 10^{7}y^{4.3}}$$

$$k(y) = 5.18 + \frac{0.2803y^{3.5}}{3.788x10^{-6} + y^{4}}$$

The overpressure function computes data for a one kiloton yield weapon. To obtain peak overpressures for other weapon yields, appropriate scaling laws are applied. A one kiloton yield is used as a basis for converting ground range and burst height values to those of any size weapon yield by using the expression $W^{1/3}$ where W is the actual weapon yield in kilotons. The expression $W^{1/3}$, which is normally called the scaling factor, is the result of actual nuclear tests and holds true for yields up to and including the megaton range (12:101). Therefore, to obtain the actual height of burst for some yield W, multiply the scaled height of burst (for one kiloton) times $W^{1/3}$. This example can also be illustrated with the following expression:

$$H = Y \times W^{1/3} \qquad (2)$$

where H is the actual burst height, Y is the scaled burst height for a one kiloton device, and W is the actual yield in kilotons. The above expression can also be used to compute scaled information from actual information by simply inverting the equation.

$$Y = H / W^{1/3} \tag{3}$$

Scaled ground range (X) can be obtained in a similar fashion by simply replacing H with the value for actual ground range, GR.

$$X = GR / W^{1/3} \tag{4}$$

Large quantities of nuclear effects data are scaled in this manner to simplify calculations. This thesis will use scaled data for a one kiloton device in all computations.

AF/SASM uses the overpressure function for numerous probability of damage (Pd) computations on their Zenith Z-150 computer system. However, since the Pd computations require range from ground zero as an output, AF/SASM has to resort to an iterative bisection routine to extract a scaled ground range value as a function of scaled height of burst and overpressure. The iterative bisection routine basically computes a series of peak overpressure values which approach a desired input overpressure value. When the desired peak overpressure value comes within a set tolerance level, the associated scaled ground range is extracted for the Pd

computations. To do a single Pd calculation, 100 ground range values are normally required (5). Each Pd calculation run takes about 30 to 40 seconds to accomplish. AF/SASM often requires 100 to 1000 Pd calculations be performed in a short period of time (6). Since the iteration slows the calculations and produces some degree of error, AF/SASM desires a more direct method of obtaining the scaled ground range as a function of scaled height of burst and overpressure. The present method is considered unsatisfactory (6).

No information was found to indicate that this problem has been solved previously. AF/SASM stated that the Defense Intelligence Agency uses the complement of a lognormal distribution to approximate the distance damage function, but it was a poor fit on a point by point basis (5). The distance damage function is the computer model used to compute the probability of damage figures. The overpressure function is a subroutine of the distance damage function and provides range inputs through the bisection routine. However, AF/SASM was unaware of any other organization working in the nuclear effects field that has determined ground range values directly as a function of overpressure and burst height (6). A review of Defense Technical Information Center (DTIC) material also showed no previous work accomplished in this area. Additionally, Harold L. Brode, the designer of the overpressure function, was

unaware of any previous work accomplished on a new expression to compute ground range directly (2).

Limitation of Scope

This thesis will examine some possible methods to solve the problem and select the one that looks most promising. No attempt will be made to improve or replace the bisection routine currently used. Although there is a large data scatter in the original atmospheric nuclear test measurements, it will be assumed that the overpressure function is accurate. This function will be used to generate any required data and evaluate the quality of any new method. The nature of the data will be covered in Chapter 2.

Definition of Terms

The information presented in this thesis comes from three major fields of study. The problem itself is related to the study of nuclear weapons effects, however, solving the problem involves mathematics and statistics. For someone not directly familiar with these disciplines, this section is provided in an attempt to make some commonly used terms more meaningful.

Overpressure is that pressure exceeding the ambient pressure, caused by the shock wave of an explosion. Peak overpressure is the maximum value of the overpressure at a given location and is generally experienced at the instant

the shock wave reaches that location (12:637). It is usually expressed in pounds per square inch (psi), however, it will also be expressed in kilopounds or thousands of pounds per square inch (ksi). Throughout this thesis, the terms P and OVP will stand for peak overpressure. This is necessary because much of the computer data was produced using the term OVP prior to the change to P.

Ground zero is that point on the surface of the earth vertically below or above the center of a burst of a nuclear weapon (12:634).

Polynomial models can best be expressed by using an example. A one-variable cubic polynomial model is written as follows:

$$X = C_0 + C_1 P + C_2 P^2 + C_3 P^3 + r \tag{5}$$

where X represents the dependent variable and P the independent variable. The C_js are the parameters that specify the nature of the relationship, and r is the error (residual), a term that takes into account the fact that the model does not exactly describe the behavior of the data. A quadratic (second degree) two-variable polynomial model is written as:

$$X = C_0 + C_1 P + C_2 P^2 + C_3 Y + C_4 Y^2 + C_5 PY + r \tag{6}$$

where Y is the second independent variable (10).

Exponential models have exponents which are variables (14:982). The overpressure function is a complicated exponential model.

The variable X is a function of P if for each value of P there corresponds only one value of X (16:205). In practice, it is denoted as $X = f(P)$. A function of two variables, $X = f(P,Y)$, is a function whose domain is a product of two sets, or is contained in a product of two sets. Mathematically speaking, there is no distinction between a function of one variable and functions of several variables (13:65).

In the study of statistics, regression analysis is used to investigate the relationship between two or more variables related in a nondeterministic fashion (9:423). This thesis uses regression analysis as a tool for curve fitting. Many of the statistical inferences normally applied to regression analysis do not pertain here. The objective of this study is not to investigate relationships between variables but to find a direct way of computing scaled ground range. For a more detailed discussion of regression, consult Neter, Wasserman, and Kutner (19).

There are numerous other statistical terms that are used in this thesis. The variance of a variable is a measure of its variability or point spread. The standard deviation is the square root of the variance and is often

used to assess how good the fit is. Devore (9) does a good job explaining most statistical terms.

The method of least squares is used by the SAS (Statistical Analysis System) computer package to do the actual curve fitting. This method generates a line that minimizes the sum of the squares of the errors (11:532). It was selected because it usually produces a good fit over a wide variety of applications. However, it is influenced by the data, placing more weight on data with higher values. This condition can therefore favor some data points over others.

Research Question and Objectives

How can the scaled ground range from ground zero be determined as a function of peak overpressure and scaled burst height given the expression for peak overpressure as a function of scaled ground range and scaled height of burst?

The primary objective in solving this problem is to find a direct method for computing the scaled ground range from ground zero as a function of peak overpressure and scaled height of burst. Secondary objectives are to insure that any new method produces values which closely match the current ones and reduces the time it currently takes to do the calculations.

II. Background

Approach

Three approaches were examined to see which would yield an accurate and reasonably non-complicated solution to the problem. They were:

A. Attempt to mathematically invert the overpressure function.

B. Build a new expression using data from the overpressure function.

C. Load a data base into the computer with an interpolation routine.

Of the three approaches, building a new expression seemed most promising for a number of reasons. The overpressure function was originally built by fitting data into a mathematical expression. A number of experts in the field, including Harold L. Brode, advised using a curve fitting approach (2). Initial attempts to mathematically invert the overpressure function by using the computerized mathematical program MACSYMA (24) showed no progress. Based on the large range of data, building a data base was considered impractical for use with AF/SASM's damage function on a Zenith Z-150 microcomputer. What AF/SASM required was an expression or series of expressions that could calculate values of ground range directly from inputs of weapon burst height and expected peak overpressures. Therefore, the

approach taken by this thesis was to build a new expression using data produced directly from the overpressure function.

To develop an equation which represents a collection of data, the following steps were necessary:

A. Collect and plot the data representing the dependent variable (X) and independent variables (Y and P).

B. Select an equation form.

C. Use some type of error-minimization regression procedure (such as the method of least squares) to determine the coefficients in the equation (18:127).

These steps are covered in this chapter. The SAS computerized statistical system was used for all regression analyses because it had a wide range of capabilities, was fairly easy to use and understand, and was readily available.

Description of the Data

The data used to build an inverse of the overpressure function was generated from the overpressure function. No attempt was made to recover the original atmospheric test data because AF/SASM desired an inverse of the overpressure function. Therefore, this function generated all the data required to build a new function. The data range spanned the following limits:

Peak overpressure (P): 1 to 3×10^6 psi

Scaled ground range (X): 0 to 8,000 ft/KT$^{1/3}$

Scaled burst height (Y): 0 to 7,000 ft/KT$^{1/3}$

The $KT^{1/3}$ in the denominator identifies these distances as scaled values (values scaled to a one kiloton yield).

The nature of the data can be best described by examining Figures 1 through 9. These curves span the range of the current fit and show the contours of constant peak overpressure plotted against scaled burst height and scaled ground range (1:60). The contours are frequently referred to as knee curves because they resemble a person's knee (12:106). The additional terms and symbols (such as Xm, Xe, or circles) in the figures are not important to this thesis.

The contours in Figure 1 show where the functional relationship between the variables fails. Along the upper portion of the knee, different values of scaled ground range can be extracted for the same combination of peak overpressure and scaled burst height. The solution in this case probably requires the partitioning of the data into two or three parts in the vicinity of the knee. This problem is not encountered with the data in Figures 2 through 9.

Also note that the peak overpressure increases exponentially as the ground range decreases. This condition can be seen clearly in Figure 10 which is a computer generated three dimensional surface of the information displayed in Figures 3 and 4. The plot shows the exponential growth of the peak overpressure for the scaled ranges .160 to .080 kft and the scaled burst heights 0 to .20 kft.

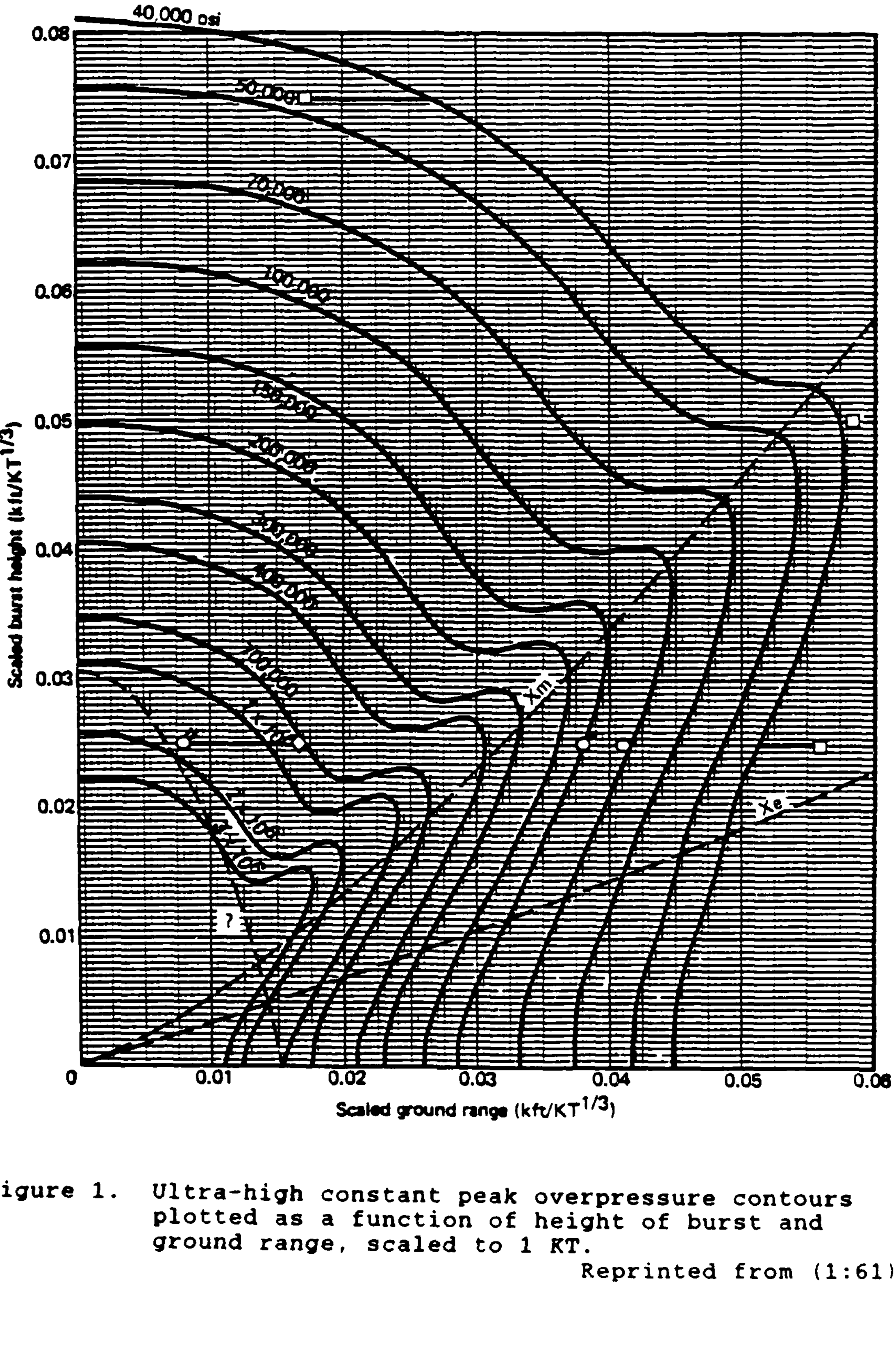

Figure 1. Ultra-high constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.

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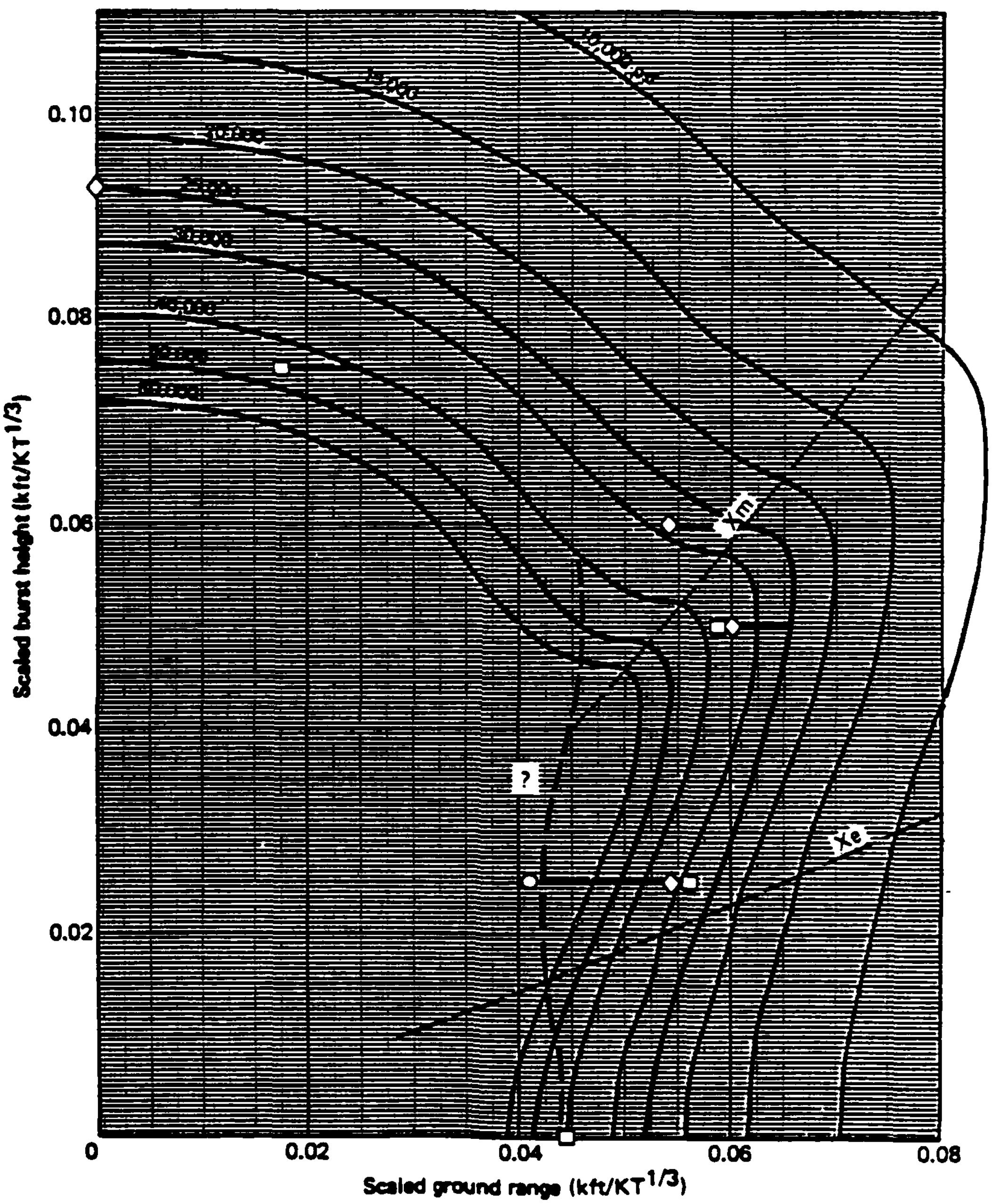

Figure 2. Extremely high constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.
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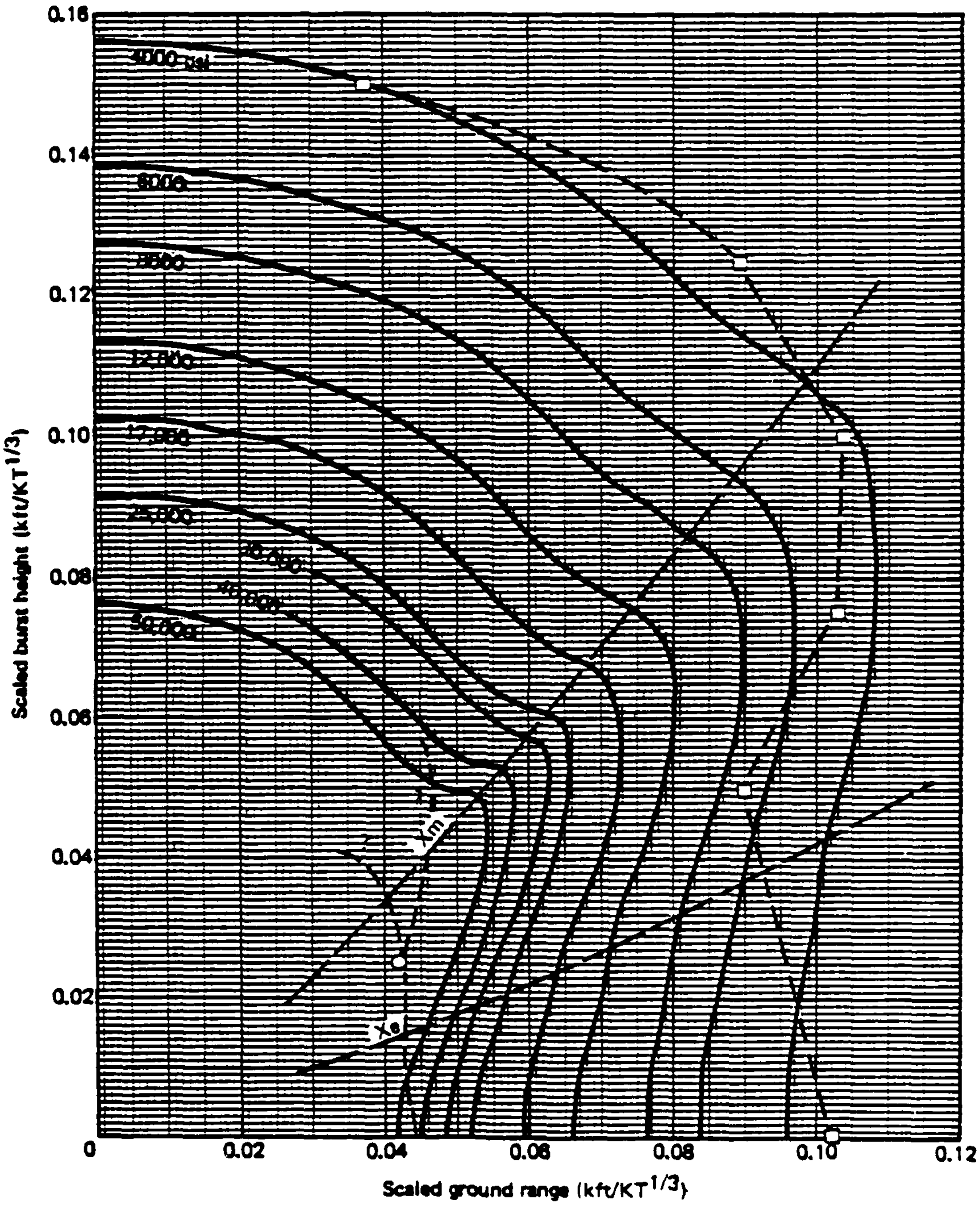

Figure 3. Very high constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.

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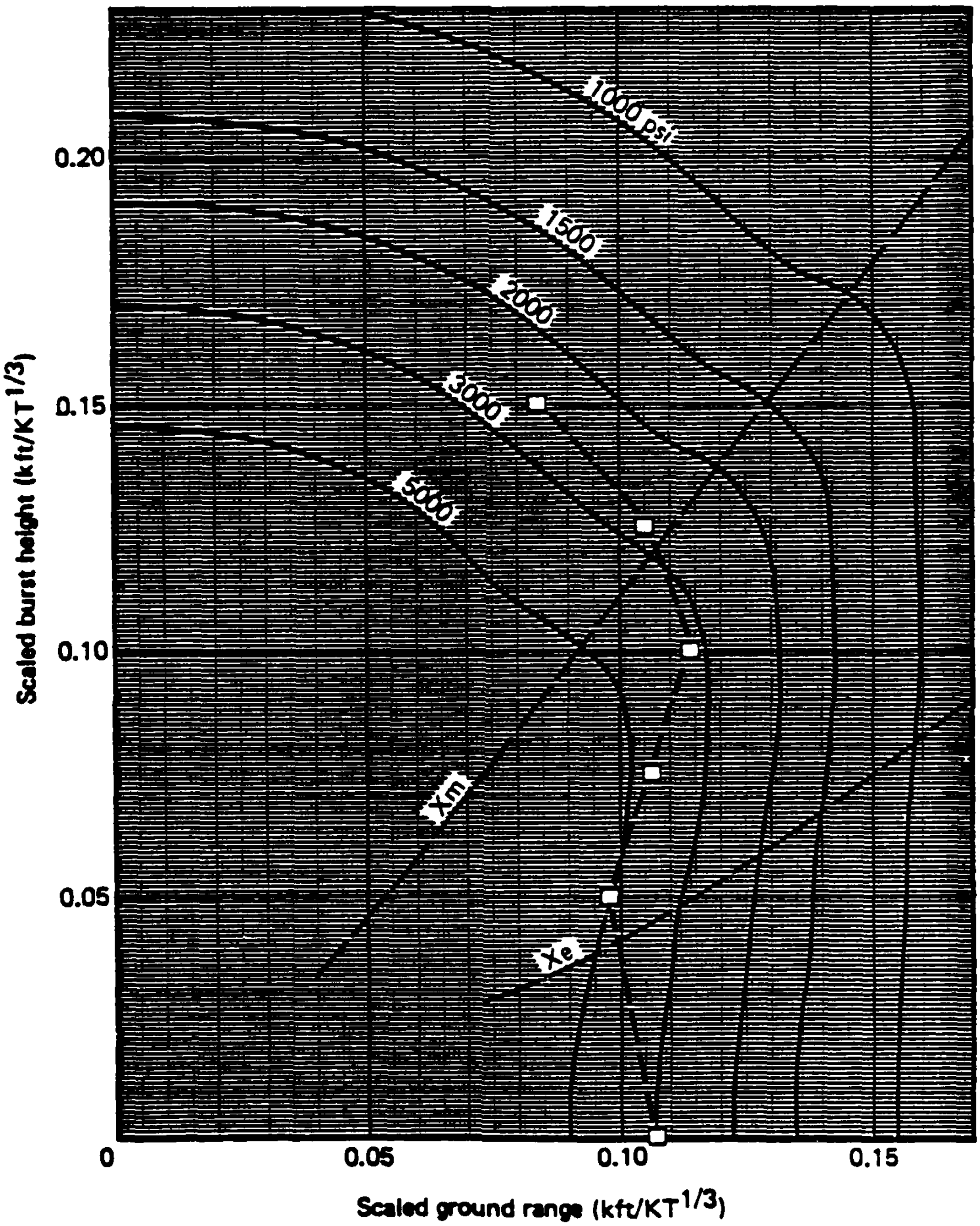

Figure 4. High constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.

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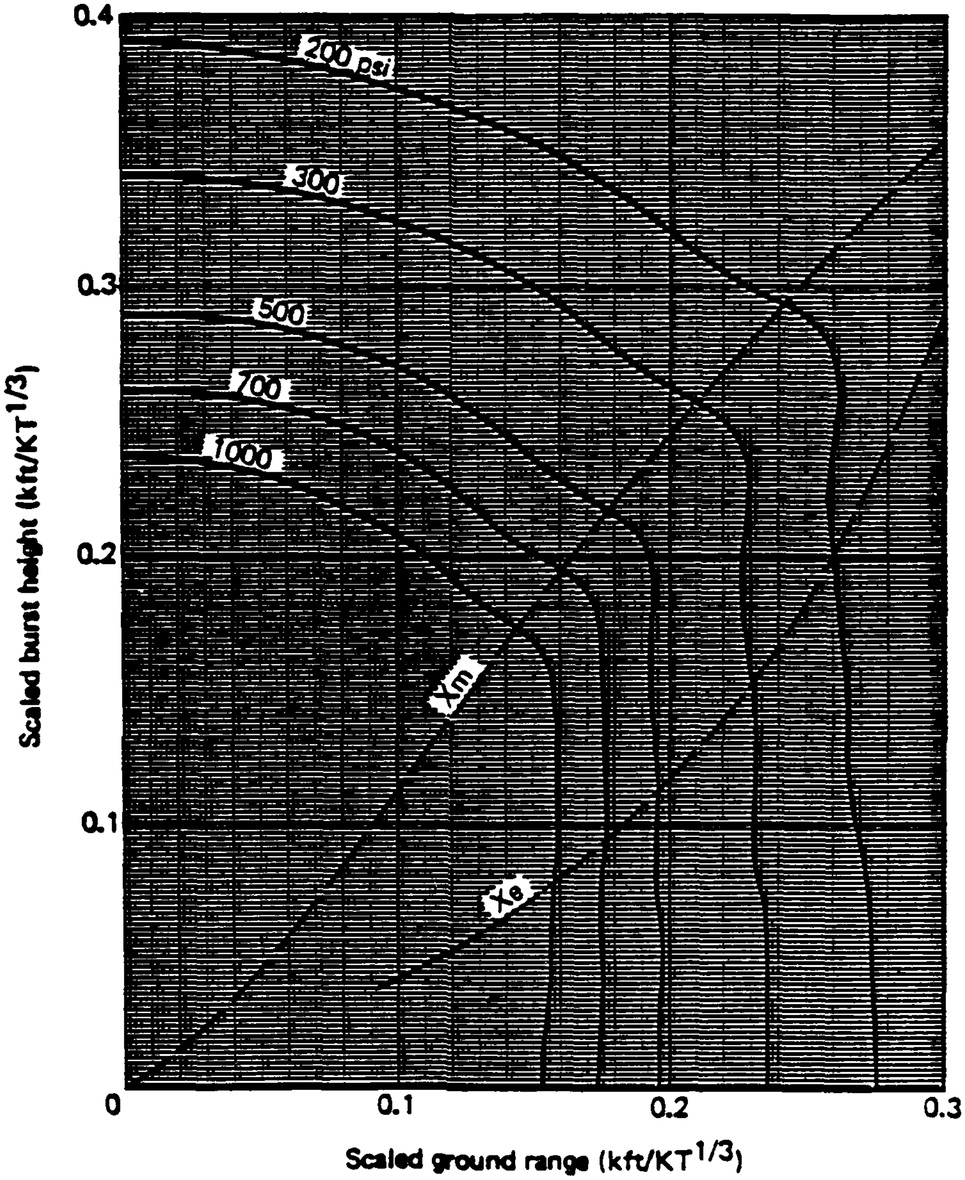

Figure 5. Intermediate high constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.
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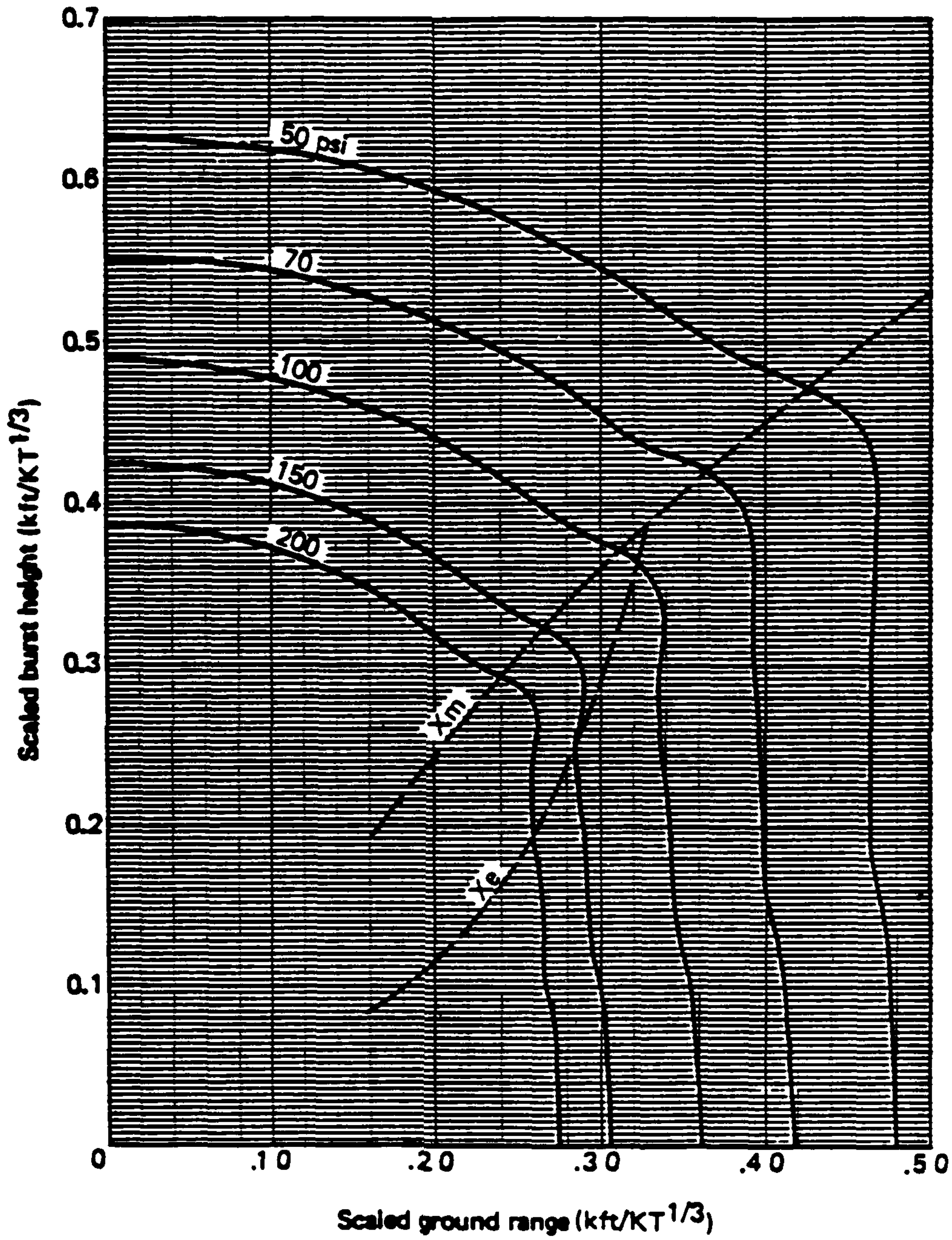

Figure 6. Intermediate constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.

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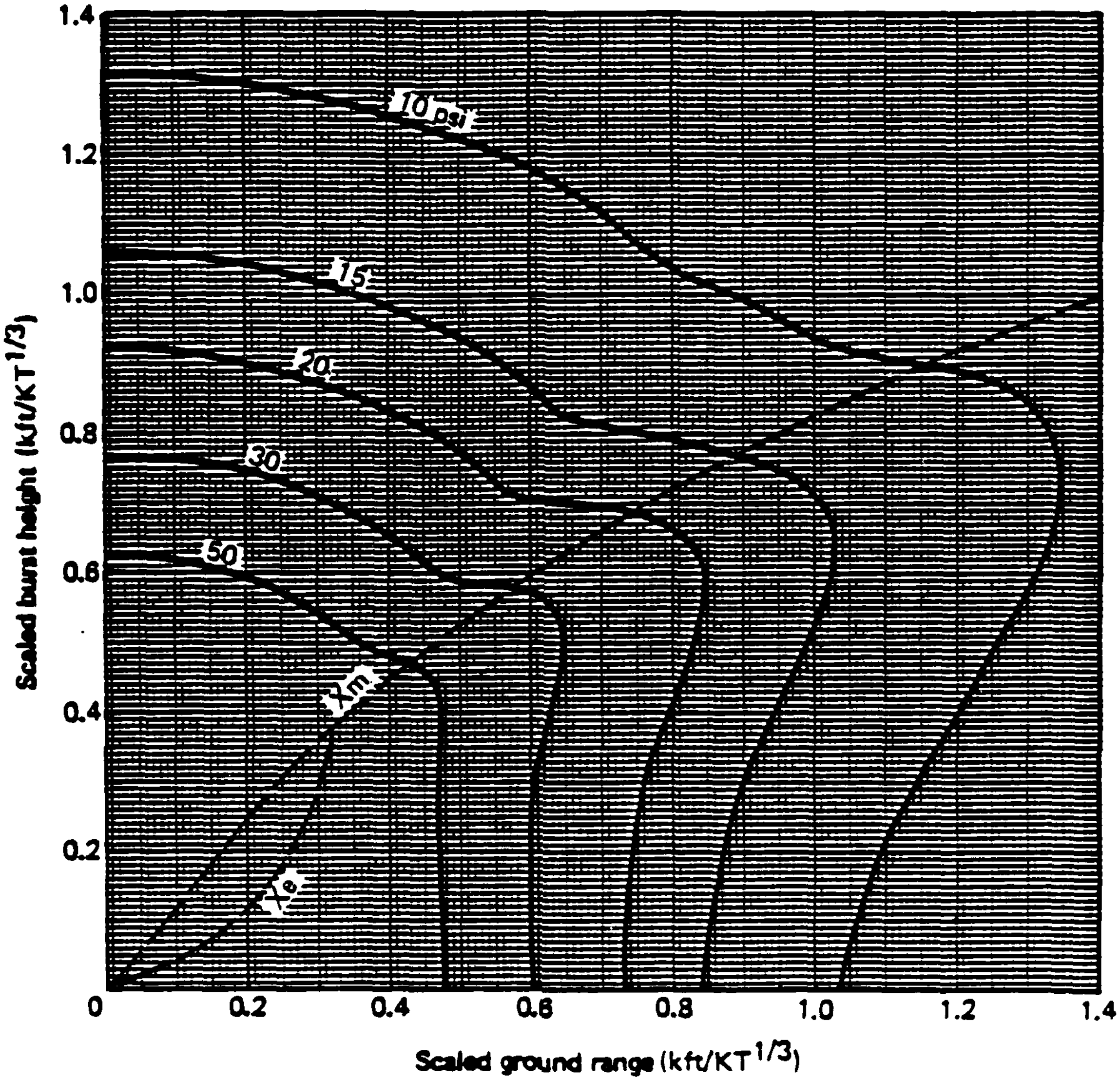

Figure 7. Intermediate low constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.

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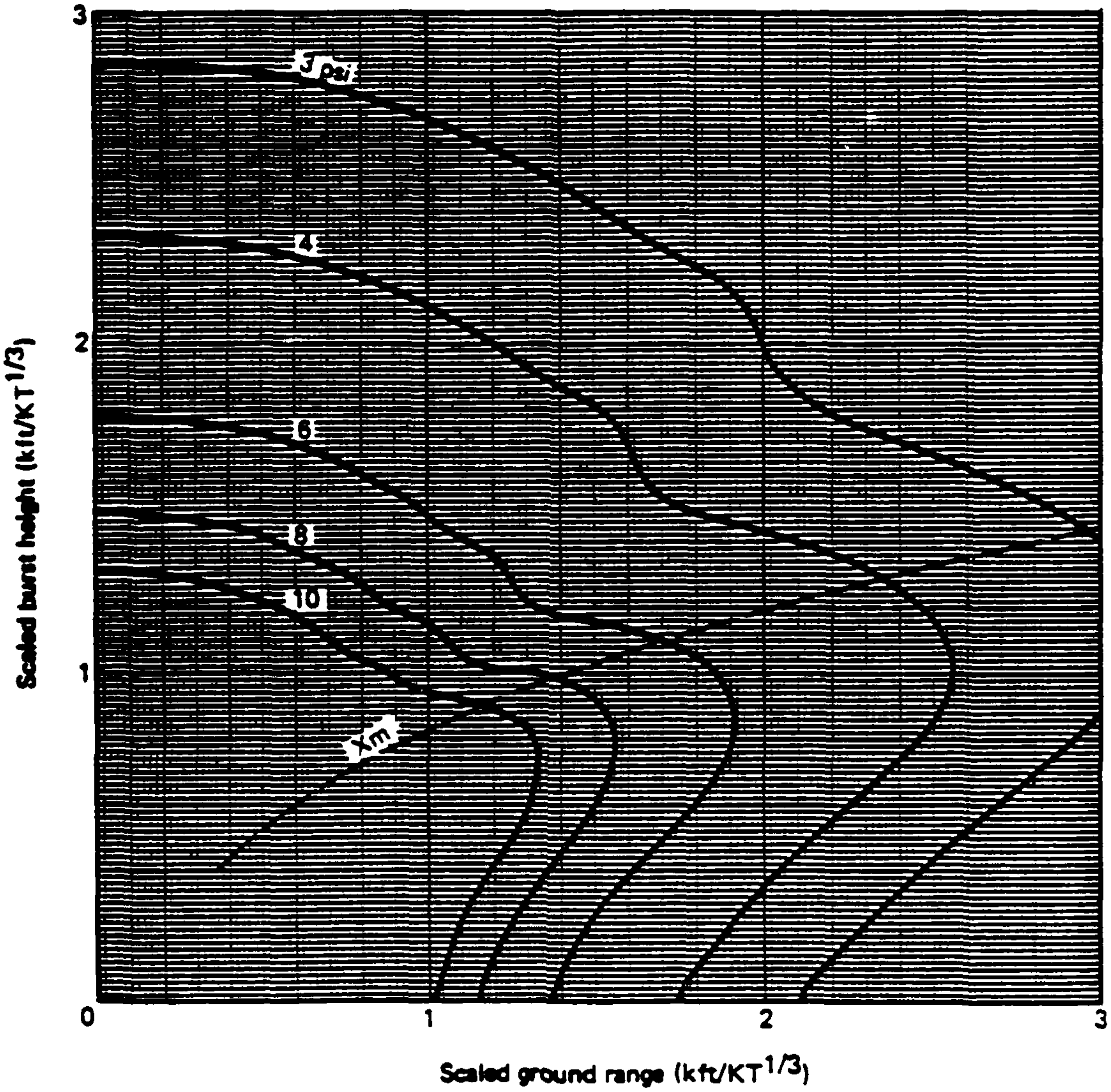

Figure 8. Low constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.

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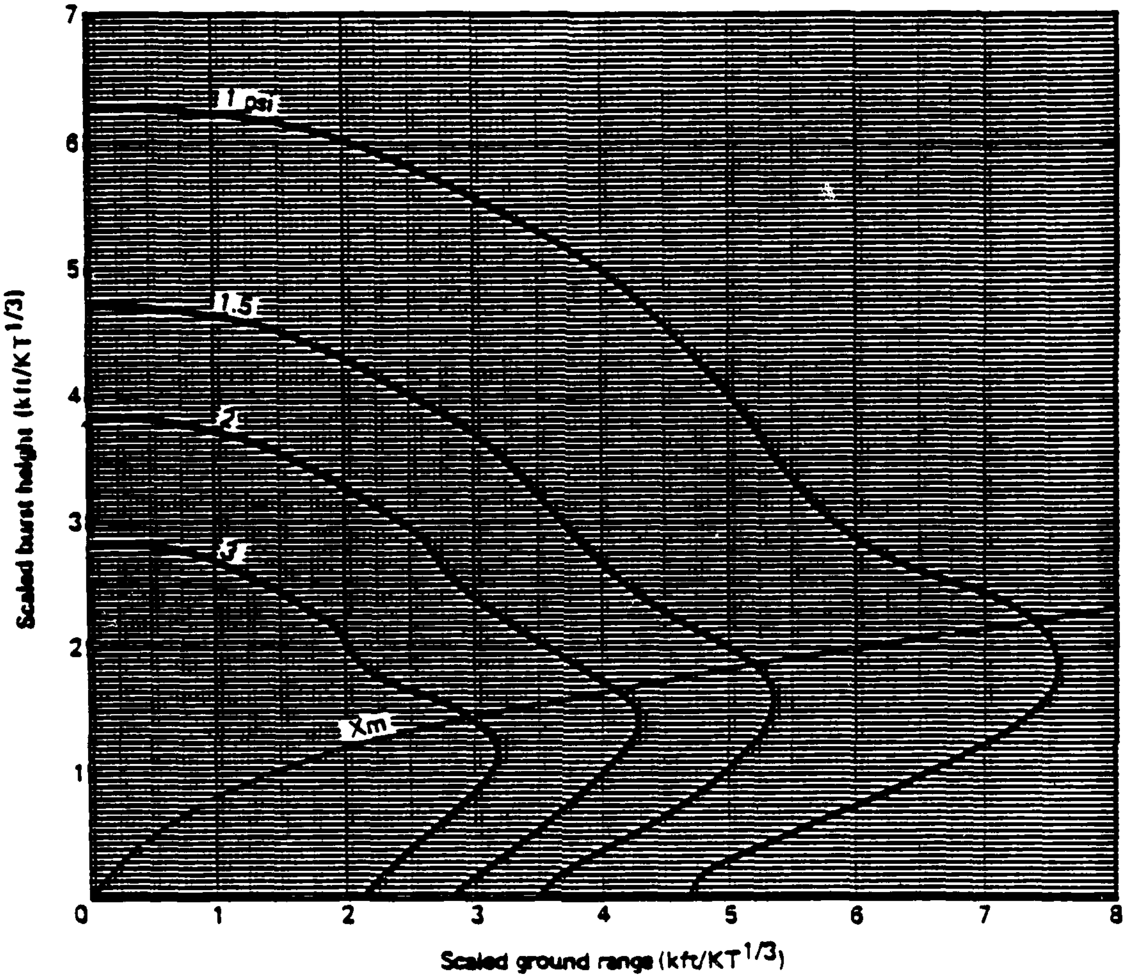

Figure 9. Very low constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.
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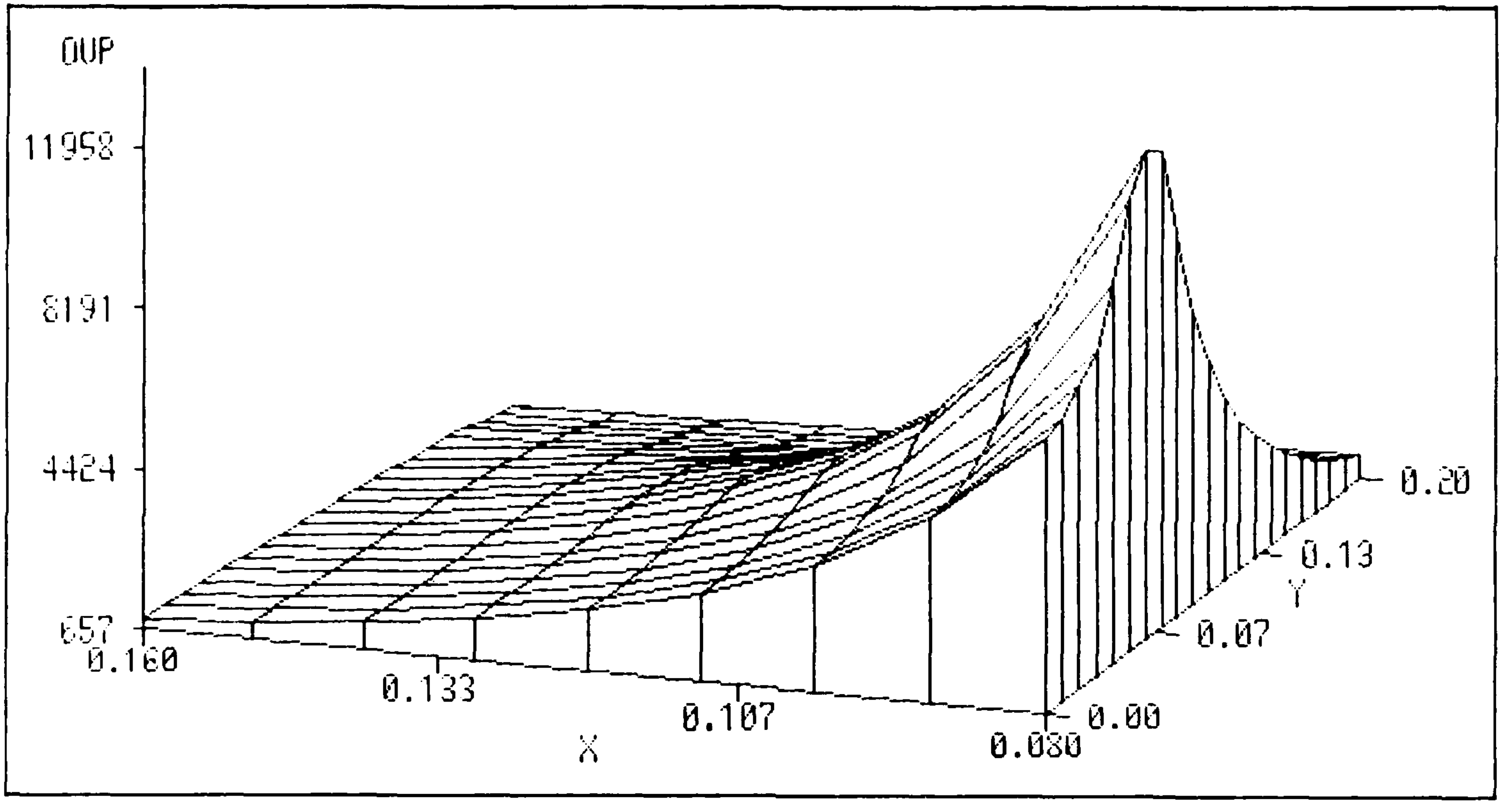

Figure 10. Three dimensional plot showing the exponential nature of peak overpressure (OVP) using data from Figures 3 and 4. OVP is in psi, X and Y are in $kft/KT^{1/3}$.

Due to the exponential nature of the peak overpressure growth and the large data range, it was decided to break the problem up into manageable segments. Since the objective was to determine the scaled ground range as a function of peak overpressure and scaled burst height, a convenient way of partitioning the data was necessary. The peak overpressure data provided a way of doing this because of its exponential nature. This data was divided into five regions: 1 to 10 psi, 10 to 100 psi, 100 to 1000 psi, 1000 to 10,000 psi, and 10,000 to 100,000 psi. These regions provided artificial bounds for the peak overpressure data. Similar bounds were defined for the scaled burst heights. The scaled burst height ranges: 0 to 6.20 kft, 0 to 1.315 kft, 0 to .488 kft, 0 to .235 kft, and 0 to .119 kft correspond to each peak overpressure region described above. Consequently, each region was defined by a specific range of data values.

The next step was to decide which peak overpressure region to use for the initial trials at solving the problem. The region selected would have to be of greatest interest to AF/SASM, since it was unclear early in the project whether there would be sufficient time to solve all the regions separately. Therefore, if the problem was solved for one region only, AF/SASM would be able to use the solution immediately in its probability of damage computer program and as a guide for solving the remaining regions. The peak

overpressure region of greatest interest was from 1000 to 10,000 psi (6).

With a starting region selected, the next step was to generate data. In certain cases, this step was a matter of simply computerizing the overpressure function and using iteration techniques to generate the quantity of data desired. A sample program for generating data can be found in Appendix B. However, because the overpressure function generates values of peak overpressure as a function of scaled ground range (X) and burst height (Y), no simple control is available to keep the values of peak overpressure within the region 1000 to 10,000 psi. To keep the data pairs of peak overpressure and burst height in this region, some of the required data was manually extracted from Figures 1 through 9 and then validated against the overpressure function routine for accuracy. A computerized bisection routine could also have performed this task.

The data sets used to accomplish the curve fitting are provided in Appendix A. The data was arranged to capture the unique qualities of the curves but limited to manageable size. Additional data was added during the curve fitting process to prevent the polynomial from wandering and to weight the data where it was sparse.

The regional data was then plotted to view the nature of the data curves. Figure 28 in Appendix C shows what the constant scaled ground range curves look like when plotted

against peak overpressure and scaled burst height. The curves in Figure 28 were computer generated, however, the same can easily be accomplished by hand on graph paper. The nonequal spacing between constant scaled ground range curves is due to the exponential nature of the peak overpressure. It is interesting to note that the curves resemble data that could be generated by polynomial equations.

Before proceeding further, some additional information must be provided about the scatter of the original atmospheric test data. Figure 11 is a graph of the approximate data scatter plotted against peak overpressure. Peak overpressure values greater than approximately 10,000 psi were generated from various calculations and do not reflect actual test data (1:60). The uppermost curve in Figure 11 roughly corresponds to a two standard deviation limit; i.e., bounding about 95 percent of the data. The lower curve approximates a one standard deviation value (1:26-29). As an example, in the 1000 to 10,000 psi peak overpressure region, the error in a range value can be as much as 100 percent. Since the variability (scatter) of the original test data is so large, any equation predicting the peak overpressure is a "best-guess" approximation and should not be assumed to be an exact predictor of the actual phenomenon. The overpressure function is not an exact predictor, but it is a good predictor of information that has much uncertainty associated with it.

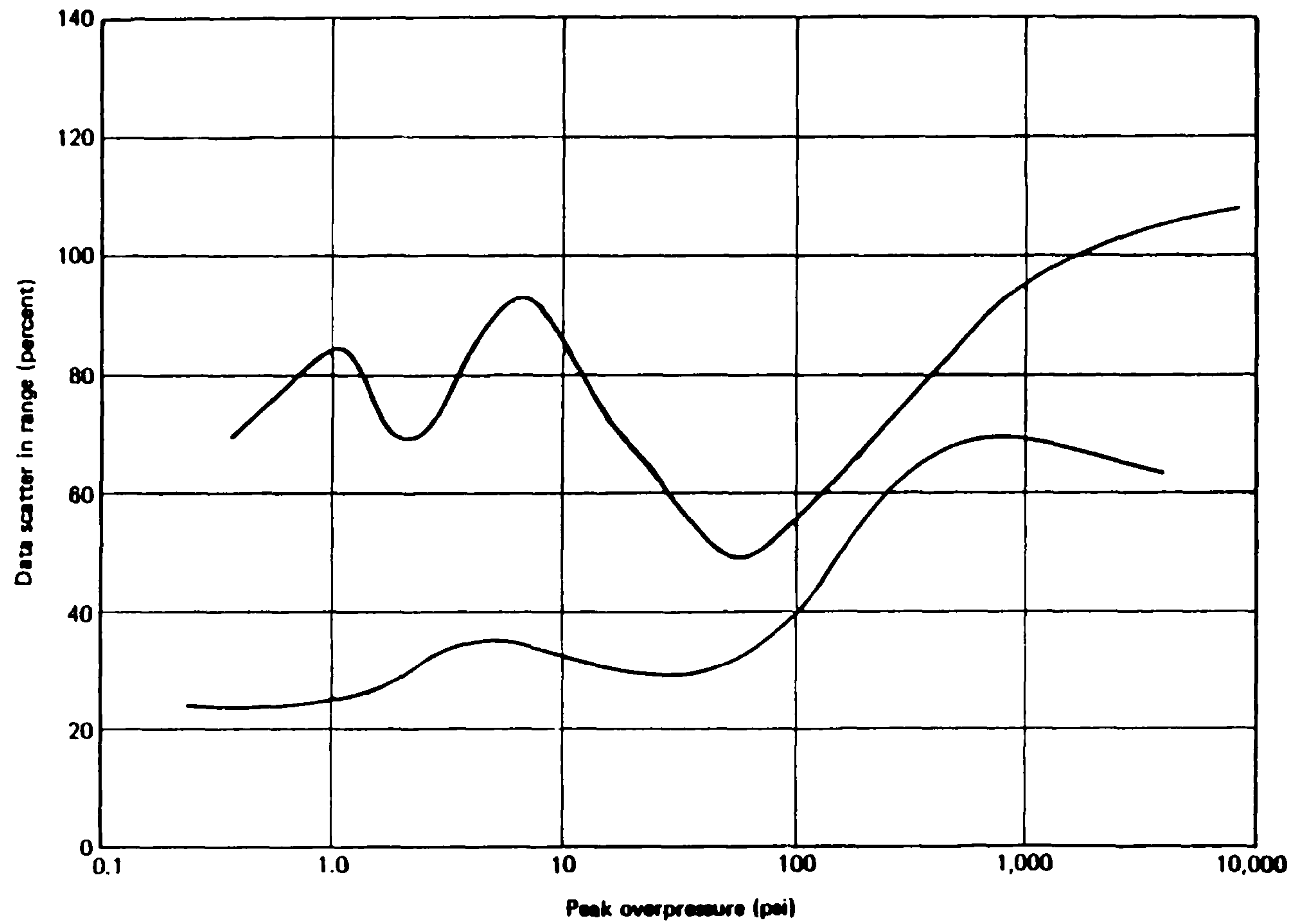

Figure 11. Scatter in percent of range to given peak overpressure for near-surface atmospheric nuclear tests. Adapted from (1:28)

Overview of the Analytical Model Selection Process

Prior to beginning the research, some level of accuracy had to be specified for the ground ranges generated by any new method. An error tolerance of plus or minus ten percent of the actual scaled ground range was initially considered a realistic figure to strive for. However, the ultimate goal was to reduce the error as much as possible.

The required model was characterized by one dependent variable (X), two independent variables (P and Y), and plotted data curves resembling a polynomial. Using previous work accomplished by Ploetner and Broadnax (20) on the same subject as a guide, a response surface approach was attempted. Since the nature of any estimated curves would result in a three dimensional surface, the response surface approach seemed like a good one. The SAS System conveniently provided a program to build two independent variable polynomial models directly by using the PROC RSREG procedure (10:110). However, this procedure was limited to a quadratic response surface model only. The results obtained did not fall within the specified error tolerance of ten percent of the actual scaled ground range values. The PROC RSREG method was dropped from further consideration because a polynomial of higher degree than two was required.

Further study revealed that there were a number of disadvantages associated with polynomial equations. Although they often fit data well, they also have a tendency

to fail if used to extrapolate data (10:139). In other words, polynomial equations are seldom effective at predicting reliable information outside the available data. Another problem associated with polynomials is that as the degree of the polynomial increases, the polynomial has a tendency to wander, sometimes excessively, in those regions not defined by data points (15:45). Consequently, a high degree polynomial must be examined closely for this wander characteristic, especially in those regions where the data points are widely spaced.

To avoid the problems associated with polynomials, a different curve fitting approach was attempted. This approach, suggested by Harold Brode (2), did not produce an accurate solution within a reasonable time period. It initially required fitting values of scaled ground range as a function of peak overpressure for constant values of burst height using a predetermined mathematical expression. The proper mathematical expression was determined through a series of trials to see if it provided a satisfactory fit to the data. Through use of an available regression package, the coefficients of the expression were determined for each specified value of burst height. Then it was a matter of finding analytic expressions for the coefficients as a function of burst height (2). This method can be illustrated through use of a second degree polynomial

expression. First solve the polynomial for each coefficient (A, B, and C) holding Y constant.

$$
\begin{array}{llllll}
\text{At } Y = 0, & X_{1,n} = A_1 & + B_1 P_{1,n} & + C_1 P_{1,n}^2 & \text{for } n = 1,2,\ldots m. & \\
\text{At } Y = 25, & X_{2,n} = A_2 & + B_2 P_{2,n} & + C_2 P_{2,n}^2 & \text{for } n = 1,2,\ldots m. & \\
\vdots & \vdots & \vdots & \vdots & \vdots & (7) \\
\text{At } Y = 200, & X_{9,n} = A_9 & + B_9 P_{9,n} & + C_9 P_{9,n}^2 & \text{for } n = 1,2,\ldots m. &
\end{array}
$$

where

m = the number of data points generated for each constant value of Y

X = the set $\{X_{1,n}, X_{2,n}, \ldots X_{9,n}\}$

A = the set $\{A_1, A_2, \ldots A_9\}$, etc.

Now find an expression for **A**, **B**, and **C** as a function of **Y**.

$$A = f_1(Y), \quad B = f_2(Y), \quad C = f_3(Y) \qquad (8)$$

Substituting the above functions into Equation (7) yields

$$X = f_1(Y) + f_2(Y)P + f_3(Y)P^2 \qquad (9)$$

The resulting expression is basically a series of functions within a function and probably the method used by Harold Brode to build the overpressure function.

This method did not produce a solution within the specified ten percent error limit. A variety of mathematical expressions were attempted with polynomials yielding the best fit over the data range. The coefficient values were so random in nature as to be almost impossible

to fit with a mathematical expression. A more detailed account of the process is covered in Appendix D.

Since polynomials provided the best fit to the given data, a method was required where the computer would fit a response surface of higher degree than two. Such a method was also available on the SAS System by using the PROC REG procedure. PROC REG is the primary SAS software procedure for performing the computations for a statistical analysis of data based on a linear regression model (10:15). It can build polynomial models with several variables if the variables are properly annotated in the DATA step. This method, which will be covered in detail in Chapter 3, has yielded a satisfactory fit for the majority of the required data.

As mentioned previously, the large data region was subdivided into the five smaller regions (Table 1) based on powers of ten of peak overpressure:

Table 1. Division of data by peak overpressure.

Region	Peak Overpressure (psi)
1	10,000-100,000
2	1,000-10,000
3	100-1,000
4	10-100
5	1-10

Region 2 was selected as the region of most significance to AF/SASM's use (6). The data from this region was then used

to build the primary model. Most discussions in Chapter 3 involve this region and its applicable model.

When a satisfactory fit for Region 2 was accomplished, the same method was used as a guide to fit the data in the other four regions. The five models were combined into a single computer program which generated ground range values for peak overpressures from 1 to 100,000 psi. This computer program was built to be used as a subroutine in AF/SASM's Probability of Damage Model (4) and replace the current overpressure function and bisection routine.

III. Methodology

Developing the Model

Model development began with the quadratic response surface model described in Chapter 2. Since the degree of the polynomial required to fit a set of data was not known in advance, higher order terms were added to the quadratic polynomial and their fit tested for accuracy (10:103). Thirty-three data points from Region 2 were used to perform this stepwise regression. Under these conditions, when the fifth degree term was added, SAS automatically set it to zero because it was a linear combination of the other variables. Since the fifth degree term did not contribute anything extra to the improvement of the overall fit at this early stage, it was decided to use a fourth degree polynomial model first.

The next step was to examine various combinations of the two independent variables. Preliminary tests showed that certain combinations contributed significantly to the improvement of the fit. It was now necessary to find all the possible combinations of two independent variables in a fourth degree polynomial. These combinations are depicted in Table 2. The regression analysis was then used to determine which variables were important to the fit and the values of their coefficients. The I in Table 2 is the intercept term.

Table 2. Possible combinations of a fourth degree, two independent variable polynomial.

	I	Y	Y^2	Y^3	Y^4
(1)	I	Y	Y^2	Y^3	Y^4
P	P	PY	PY^2	PY^3	PY^4
P^2	P^2	P^2Y	P^2Y^2	P^2Y^3	P^2Y^4
P^3	P^3	P^3Y	P^3Y^2	P^3Y^3	P^3Y^4
P^4	P^4	P^4Y	P^4Y^2	P^4Y^3	P^4Y^4

Since there were now 24 variables to evaluate, a better approach was to use a regression analysis to evaluate all the variables first and then eliminate those variables which did not contribute significantly to the fit rather than steadily adding variables to the equation. A sample program to accomplish this regression using PROC REG is in Appendix B. Peak overpressure data was converted to Ksi to avoid computing the squares of very large numbers and to make the numbers comparable to X and Y which are in Kilofeet.

The results are depicted in Figure 12. The circled numbers have been added to the output to key the descriptions that follow.

1. The name of the dependent variable is X (10:19).
2. The mean squares are the corresponding sums of squares divided by their respective degrees of freedom. The MEAN SQUARE for ERROR (.00002649913) is an unbiased estimate of the variance of the error (10:19). This value was used as the governing criterion for best fit.
3. The ROOT MSE of .005147731 is the square root of the MEAN SQUARE ERROR and estimates the standard deviation of the residuals (10:19). In this case,

DEP VARIABLE: X (1)

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	24	0.18296418	0.007623507	287.689	0.0001
ERROR	98	0.002596915	.00002649913 (2)		
C TOTAL	122	0.18556109			

(3) ROOT MSE	0.005147731	R-SQUARE (4)	0.9860
DEP MEAN	0.08748293	ADJ R-SQ (5)	0.9826
C.V.	5.884269		

PARAMETER ESTIMATES

(6) VARIABLE	DF	(7) PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	(8) PROB > |T|
INTERCEP	1	0.19927367	0.01653322	12.053	0.0001
OVP	1	-0.0598059	0.01896395	-3.154	0.0021
Y	1	0.34141995	1.11112219	0.307	0.7593
PY	1	0.03326228	1.32696015	0.025	0.9801
PSQ	1	0.01281487	0.00661851	1.936	0.0557
P2Y	1	-0.0389182	0.47615723	-0.082	0.9350
YSQ	1	-7.06819	21.79768674	-0.324	0.7464
Y2P	1	-3.64861	27.18512535	-0.134	0.8935
PYSQ	1	2.25195648	10.09285975	0.223	0.8239
PCUB	1	-0.00130086	0.0008923109	-1.458	0.1481
P3Y	1	0.006871166	0.06594397	0.104	0.9172
P3Y2	1	-0.35025	1.45011224	-0.242	0.8096
YCUB	1	26.46917386	154.78008701	0.171	0.8646
Y3P	1	91.40607742	201.99667142	0.453	0.6519
PYC	1	5.11704579	11.94212976	0.428	0.6692
PQT	1	.00004885307	.00004061469	1.203	0.2319
P4Y	1	-0.000358384	0.003080612	-0.116	0.9076
P4Y2	1	0.01740965	0.07029175	0.248	0.3049
P4Y3	1	-0.247744	0.61041287	-0.406	0.6857
YQT	1	55.14263313	361.80385196	0.152	0.8792
Y4P	1	-454.693	497.09169442	-0.915	0.3626
Y3P2	1	-35.9761	78.67412170	-0.457	0.6485
Y4P2	1	136.82678161	208.53526081	0.656	0.5133
Y4P3	1	-19.281	34.33685867	-0.562	0.5757
PYQT	1	0.94920871	1.87879918	0.505	0.6145

Figure 12. PROC REG output using all possible combinations of a fourth degree, two independent variable polynomial.

the overall average error of the fit is .0051 kilofeet or 5.1 feet.

4. The R-SQUARE is the MODEL SUM OF SQUARES divided by the TOTAL SUM OF SQUARES (10:19). A high value of R-SQUARE indicates that a major portion of the variation of X is due to the variation of the independent variables in the model (10:23). A high value usually denotes a good fit.

5. The ADJ R-SQ measures the reduction in mean square due to regression. It is also used to overcome the objection that R-SQUARE is a poor measure of goodness of fit because it can be forced to fit something perfectly by adding superfluous variables to the model. ADJ R-SQ tends to stabilize to a certain value when an adequate set of variables is included (10:23).

6. The VARIABLE heading matches the computed coefficients with their variable (10:20).

7. The values in the column labeled PARAMETER ESTIMATE are the estimated coefficients (10:20).

8. The column headed by PROB > |T| gives the estimated P values for the adjacent t statistics (10:20). It is sufficient to say that if a value in this column is large (e.g., greater than .5), there is a good probability that the coefficient is equal to zero and its variable can be eliminated from the equation.

Only sufficient information was presented here to explain the curve fitting process. Freund and Littell (10) should be consulted if further guidance is desired.

The curve fitting procedure required the removal of the variable with the highest P value and then accomplishing a regression analysis of the remaining variables. If the MEAN SQUARE (MS) ERROR continued to decrease, another variable with the highest P value was removed and the regression analysis was again accomplished on the remaining variables. This backward elimination process was continued until the MS

ERROR began to increase. The process was then stopped, and the set of variables which produced the lowest MS ERROR was selected as the model. This iterative process is displayed in Table 3.

Table 3. Variable elimination process using lowest MS ERROR value as the stopping criterion.

TRIAL	TOTAL ESTIMATES	MS ERROR	ADJ R^2	VARIABLE REMOVED
1	25	.0000265	.9826	---
2	24	.0000262	.9828	PY
3	23	.0000260	.9829	P4Y
4	22	.0000257	.9831	YQT
5	21	.0000255	.9832	P3Y
6	20	.0000253	.9834	Y2P
7	19	.0000251	.9835	P4Y2
8	18	.0000248	.9837	P3Y2
9	17	.0000247	.9838	P2Y
	STOP			
10	16	.0000248	.9837	PYSQ

Removing the PYSQ variable increased the MS ERROR. Therefore, this variable was considered a valid contributor to the model while all the previous variables were rejected as not contributing significantly to the fit.

The resulting fit is provided by the output in Figure 13. The contributing variables are in the left column while the coefficients of the variables are in the column entitled PARAMETER ESTIMATE. The P values are all reduced in size compared to those of Figure 12 because each variable makes a significant contribution to the fit. The final model takes the form:

SAS 1
20:28 SATURDAY, JANUARY 23, 1988

DEP VARIABLE: X

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	16	0.18294449	0.01143403	463.198	0.0001
ERROR	106	0.002616609	.00002468499		
C TOTAL	122	0.18556109			

ROOT MSE	0.004968399	R-SQUARE	0.9859
DEP MEAN	0.08748293	ADJ R-SQ	0.9838
C.V.	5.679278		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	0.20154762	0.007240166	27.837	0.0001
OVP	1	-0.0621137	0.007796939	-7.966	0.0001
Y	1	0.26548571	0.12080940	2.198	0.0302
PSQ	1	0.01362242	0.002620167	5.199	0.0001
YSQ	1	-7.29235	2.49774969	-2.920	0.0043
PYSQ	1	0.07036665	0.05523407	1.274	0.2055
PCUB	1	-0.00141394	0.0003466093	-4.079	0.0001
YCUB	1	39.26591463	10.53963454	3.726	0.0003
Y3P	1	47.69269458	10.25632356	4.650	0.0001
PYC	1	1.41551464	0.64193147	2.205	0.0296
PQT	1	.00005402714	.00001567345	3.447	0.0008
P4Y3	1	-0.0671128	0.03681966	-1.823	0.0712
Y4P	1	-314.529	54.32858738	-5.789	0.0001
Y3P2	1	-12.1885	3.59664733	-3.389	0.0010
Y4P2	1	64.01100943	24.73808956	2.588	0.0110
Y4P3	1	-8.28738	4.96603625	-1.669	0.0981
PYQT	1	0.42330598	0.29548361	1.433	0.1549

Figure 13. Optimal output variables and coefficients for a fourth degree polynomial model of Region 2 data.

$$X = .20154762 - .0621137P + .26548571Y + .01362242P^2 - 7.29235Y^2 + .07036665P^2Y^2 - .00141394P^3 + 39.26591463Y^3 + 47.69269458Y^3P + 1.41551464P^3Y^3 + .00005402714P^4 - .0671128P^4Y^3 - 314.529Y^4P - 12.1885Y^3P^2 + 64.01100943Y^4P^2 - 8.28738Y^4P^3 + .42330598P^4Y^4 \quad (10)$$

where

X = scaled ground range in kilofeet

Y = scaled burst height in kilofeet

P = peak overpressure in kilopounds per square inch

The variables match those in Table 2.

Equation (10) produced the results in Figure 22 of Chapter 4. The curves closely approximate the original data. A comparison of these predicted data curves to actual data curves can be found in Appendix C.

Validating the Model

This section covers an analysis of the model (Equation (10)) to insure that it is a close approximation of the overpressure function for the region encompassing 1000 to 10,000 psi peak overpressure. Although an exact fit is most desired, this condition is rarely attainable when using approximating methods. The next best outcome is for the approximation to produce a set of data that is within some error tolerance. For this region of peak overpressure values, the error was initially specified as plus or minus

ten percent of the ground range. This figure seems reasonable except for the fact that the ground range varies from some predetermined value for a surface burst (burst height equals zero) to a value of zero at a sufficiently high burst height as reflected in Figures 1 through 9. Ten percent of a ground range close to zero was considered a very small error tolerance indeed.

A better method was to examine each error (residual) on a point by point basis to see which data points were poor fits. Since the model was developed for use by AF/SASM, it was also considered necessary to determine what error they were willing to accept. AF/SASM personnel specified a requirement for a good fit (within ten percent) in the region from the surface to and including the characteristic knee. Beyond the knee, the error tolerance could be relaxed; however, the model still had to basically follow the characteristics of the actual data produced by the overpressure function (7). In other words, unexplained deviations were not considered acceptable.

The use of standard statistical testing was considered inappropriate for this problem because some of the underlying assumptions such as equality of variance were not present. Also, a problem with classical hypothesis-testing theory is that it regards a model as either true or false. In practice, most models are neither true nor false. They

are simply approximations to reality developed for a specific purpose (17:167).

Residual analysis is best accomplished graphically. SAS provides programming options that perform this function. The residuals, which are the difference between the actual data and the data generated by the model (predicted data), are often plotted against the predicted data. Figure 14 shows such a plot. The letter A signifies there is just a single observation at that point, while the letter B signifies two observations at the same point, etc.

The standard deviation is often used to judge how good the fit is. From Figure 13, the standard deviation was approximately equal to .005. This value equates to five feet of error in reality. The majority of acceptable data usually falls within plus or minus two standard deviations which is where most of the data falls in Figure 14. However, there are some residuals that have a higher error (up to .015), and these residuals must be examined to see if they adversely impact the model.

A difficulty with estimated residuals is that they are not all estimated with the same precision. More precise residuals called studentized residuals are obtained by dividing each residual by its standard error (10:49). Figure 15 is a plot of the studentized residuals. Its characteristic appearance is similar to Figure 14. Statistically, studentized residuals identify those

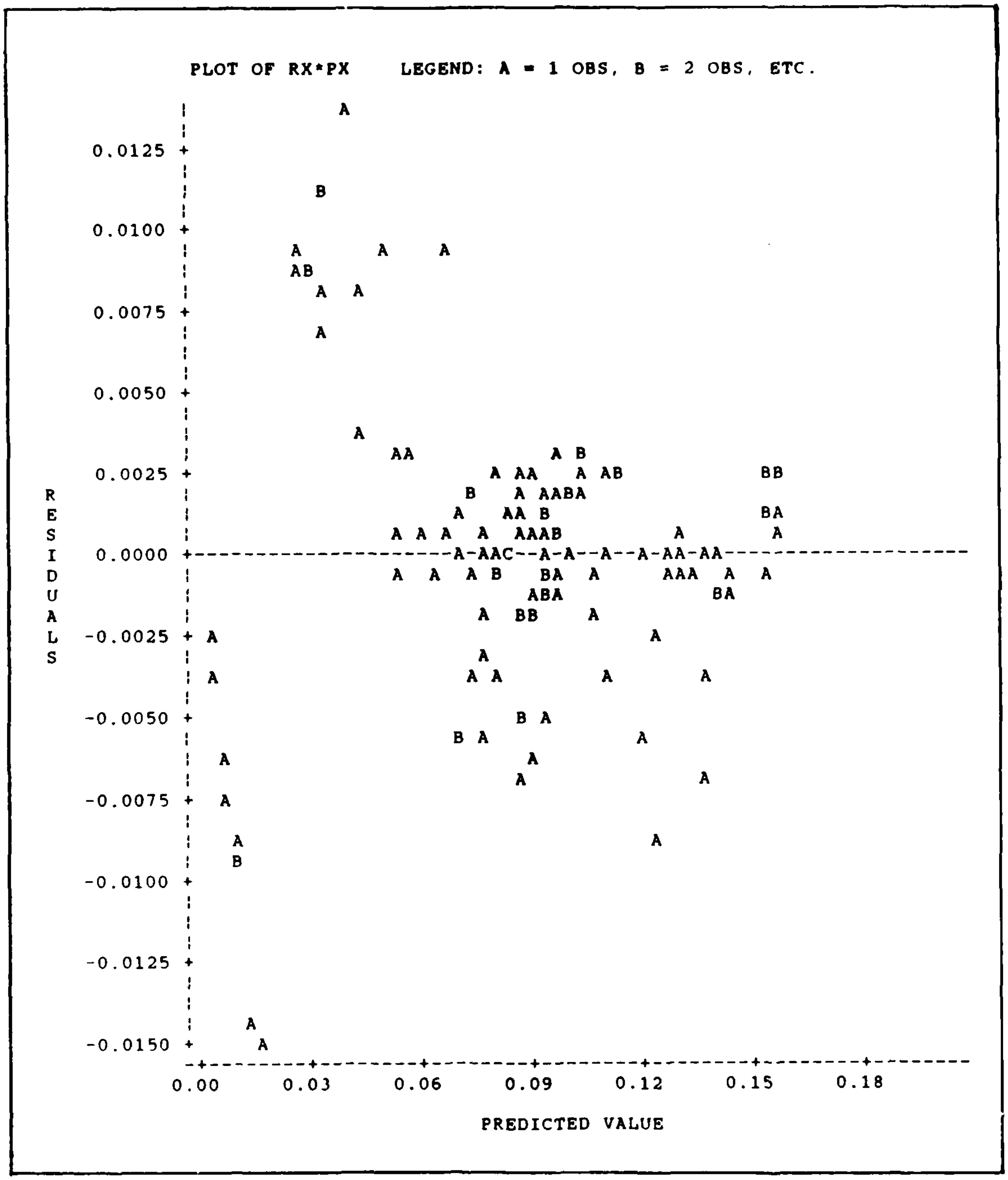

Figure 14. SAS plot of residuals (kft) versus predicted values of X (kft) for the peak overpressure region of 1000 to 10,000 psi.

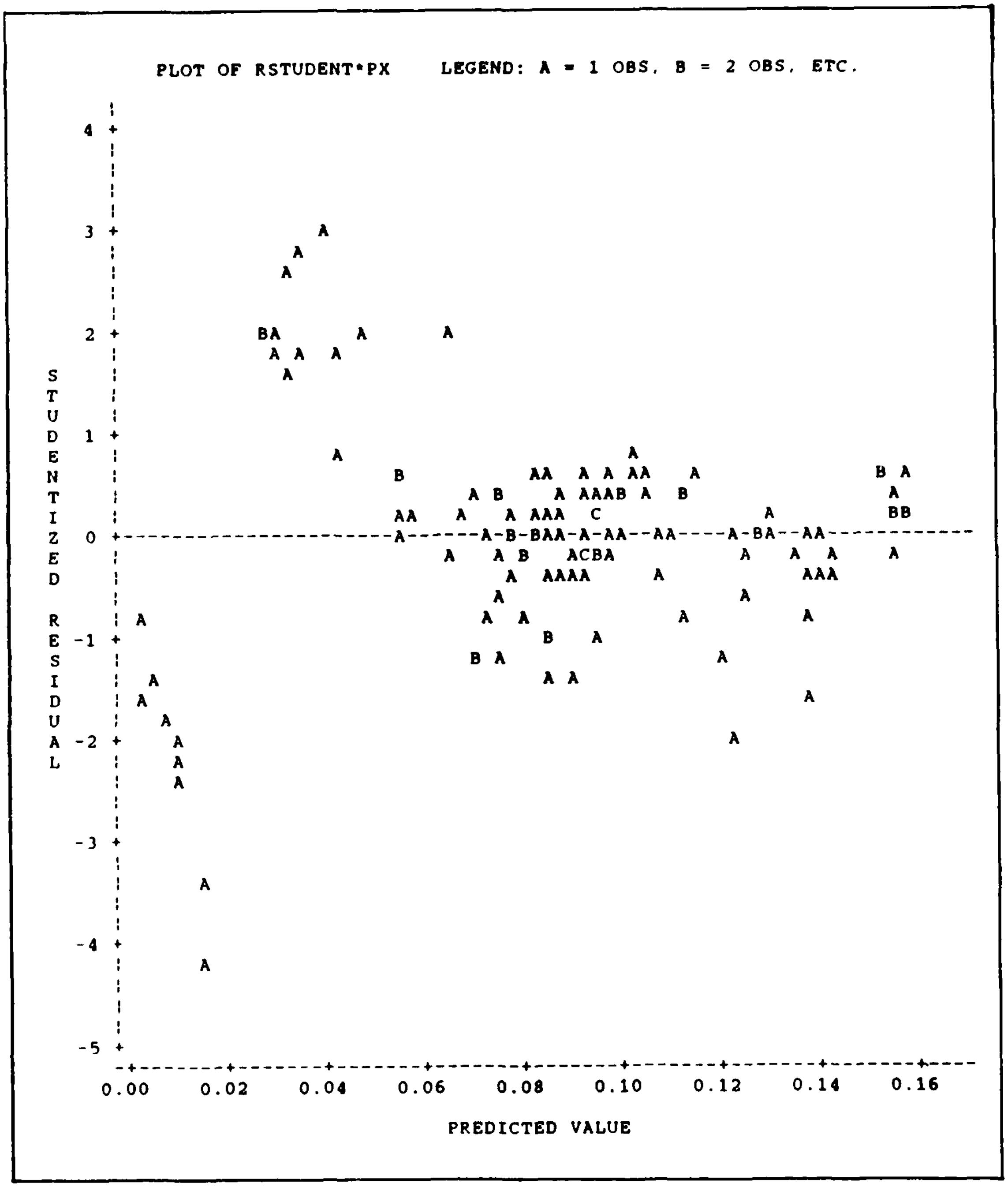

Figure 15. SAS plot of studentized residuals versus predicted values of X (kft) for the peak overpressure region of 1000 to 10,000 psi.

observations that do not appear to fit the model. For experimentally derived data, these observations or outliers can be significant. For this analysis, they simply reflect the variability of the fit.

Plotting the residuals against corresponding values of peak overpressure (Figure 16) and burst height (Figure 17) provide some interesting information. The worst fit occurs in the region of low peak overpressure and high burst height. This region also corresponds to data beyond the knee where values of ground range rapidly drop off to zero with increasing burst height. Since this region was not significant to AF/SASM's use, the large amount of error (up to 15 feet) was considered acceptable.

Three attempts were made to improve the fit and reduce the error of the region beyond the knee. The first method involved placing a greater weight on low values of X for the regression. This method reduced the residuals of the low values of X substantially at the expense of the remaining data. The resulting overall fit was worse than the original fit.

The second method involved adding eleven more data points to the poorly fitted region with the intention of forcing a fit. Starting the regression analysis from the beginning using 24 variables, two interesting things occurred. The final fit had the exact same variables as previous fits while the coefficients changed as expected,

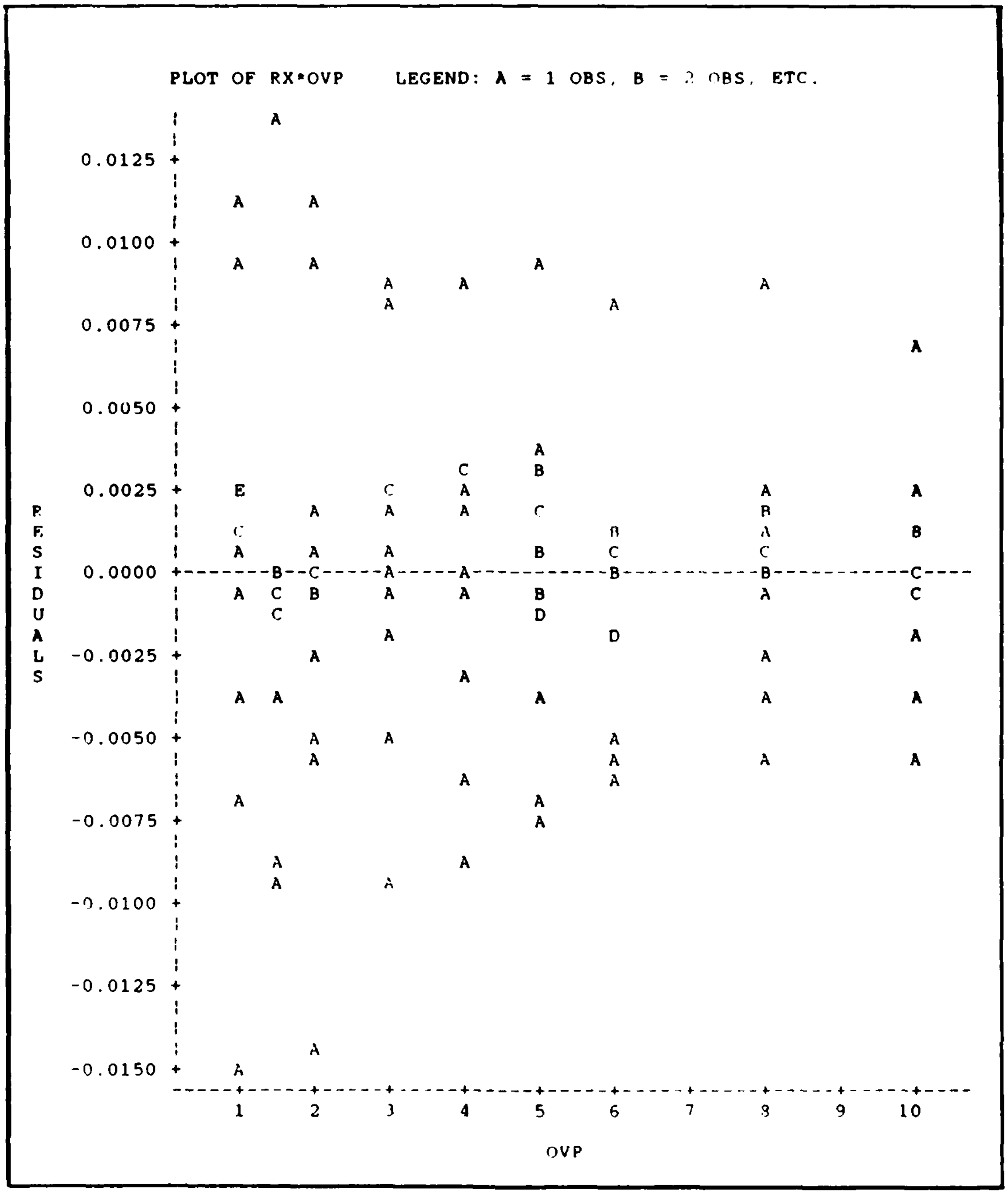

Figure 16. SAS plot of residuals (kft) versus actual values of peak overpressure (ksi).

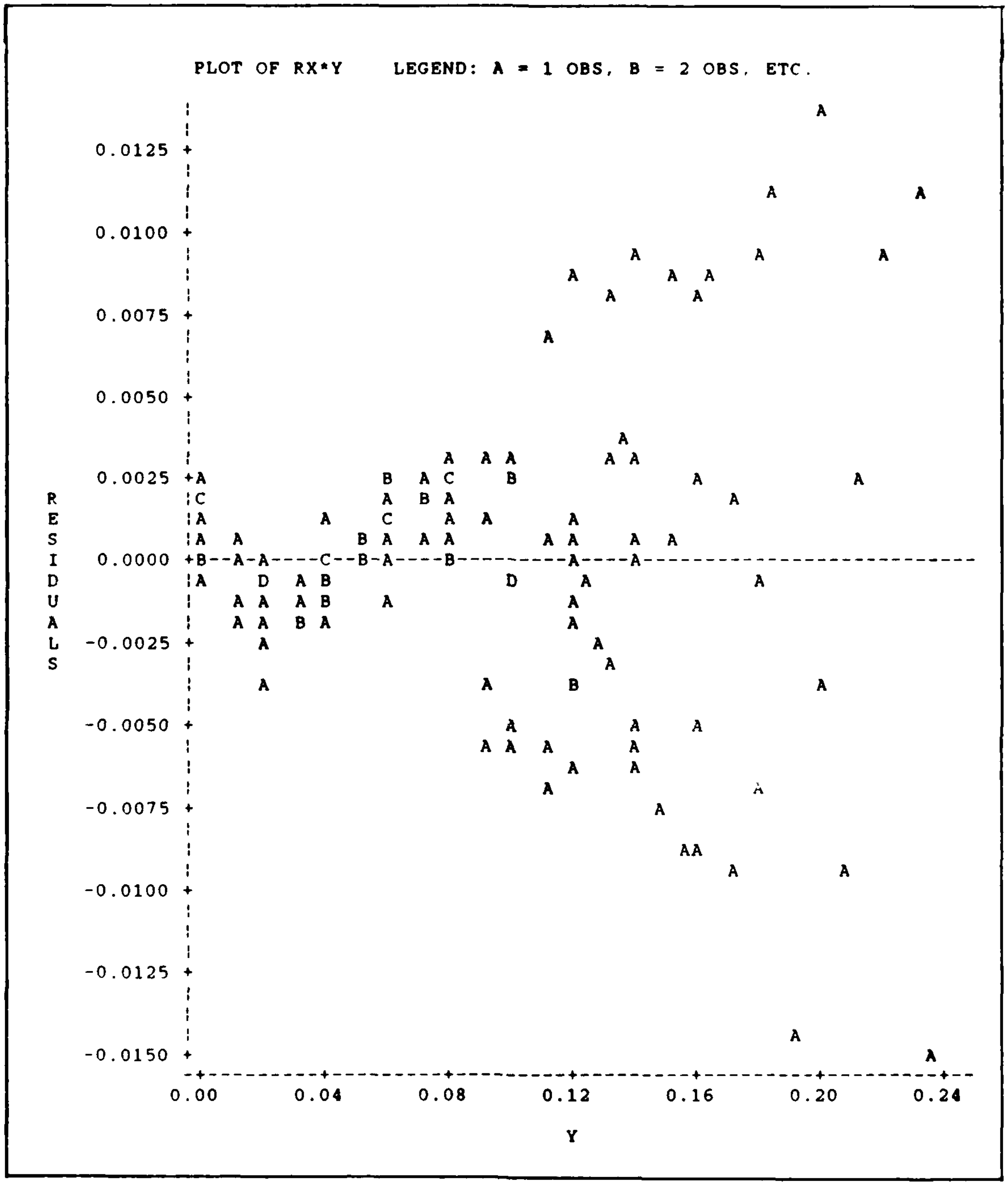

Figure 17. SAS plot of residuals (kft) versus actual values of scaled burst height (kft).

and the error remained virtually unchanged. This result demonstrated that the fourth degree polynomial model was consistent for the fitted region and that adding more data would not change its variables. Any change in data, however, changed the values of the coefficients to a minor degree. Adding the data to the region of poorest fit also increased the overall variance because more high error terms were added. This condition implied a capability to improve the appearance of the output (variance and standard deviation) by manipulating the data.

The third method employed in the attempt to improve the fit was to break the data into two parts, 1000 to 4000 psi and 4000 to 10,000 psi, and fit each part individually. The results also did not show an improvement over the original fit.

The model was very sensitive to changes in the coefficients. That is why the coefficients are usually large. Attempts to round off the coefficients generated by SAS resulted in unexpected errors. Therefore, extreme care must be used when entering these coefficients into a computer program to prevent an erroneous digit or a misplaced decimal from ruining the approximation.

The polynomial computes information for any inputs of peak overpressure and burst height, correct or incorrect, because it is simply an equation. Beyond the $X = 0$ contour, the values of X computed by the polynomial become

negative. In reality, a negative range does not exist, however, it is important to understand how the polynomial behaves beyond the data set. This way, unexpected results can be avoided.

A large uniformly spaced linear data set was generated by the polynomial to access the presence of possible wobble where data was lacking. No evidence of wobble was found from visual inspection or a graphical plot of the data.

The predicted data generated by the model closely matched the actual data in the area of interest. To prove this point, contours of equal peak overpressure from predicted data were plotted over contours of equal peak overpressure from actual data. The closeness of the approximation can be seen in Figure 18. The dotted contours represent the predicted data. The contours of actual data are the same as those found in Figures 3 and 4.

The graphics used throughout this study were an extremely valuable tool in assessing the nature of the predicted data. Contour graphs provided proof that the data was valid and caught subtle errors caused by the polynomial due to lack of data in a specific area. This occurrence of error identification was especially true during the initial curve fitting of Regions 1, 3, and 4 where large spacing of peak overpressure data allowed the curves to wander. The addition of data usually corrected any problems associated with wander. It should also be noted that the graphics are

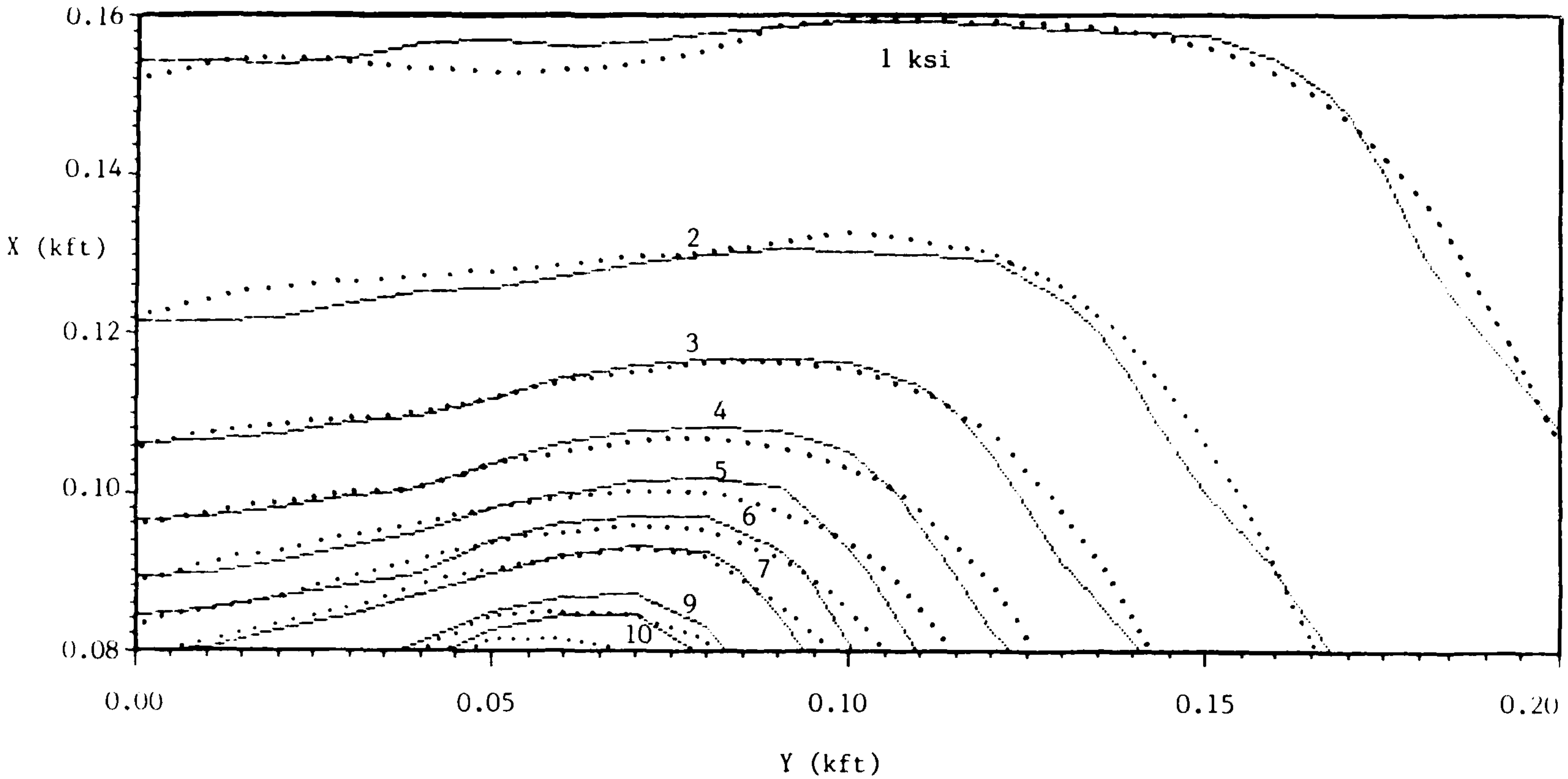

Figure 18. Comparison of predicted (dotted) versus actual constant peak overpressure contours plotted as a function of height of burst and ground range, scaled to 1 KT.

subject to some error due to interpolation errors in regions lacking data. However, this error is small and does not detract from the usefulness the graphics provide when making comparisons between predicted and actual data.

Accomplishment of Research Objectives

The previously described method was used to build similar approximations for the remaining regions. Regions 4 and 5 required the use of fifth degree terms because a fourth degree polynomial did not provide a satisfactory fit. These five fits (ground range functions) span a peak overpressure range from 1 psi to 100,000 psi. This range is still less than that covered by the overpressure function. The overpressure function computes peak overpressures to 3×10^6 psi. This high limit was not considered necessary by AF/SASM for their probability of damage calculations (8).

The five ground range functions were combined into a computer program using Fortran 77 programming language. The program can be found in Appendix B. This computer program, though built to stand alone, can easily be modified into a subroutine for inclusion into AF/SASM's Damage Function. Because the computer program is limited to the peak overpressure range stated above, a built in check is included to prevent calculating ground ranges outside these limits. This check prevents someone from using the polynomial to extrapolate data outside the data range since polynomials often fail at this task. Another check is

provided to set the ground range (X) equal to zero when the polynomial computes a negative ground range value. It is important to note that for Region 5, the values of X do not continue into the negative region but swing back up and become positive again. This condition is graphically displayed in Figure 19 and occurs only in the vicinity of 9.9 psi peak overpressure and 2,400 ft burst height. It is outside the realm of the data and is mentioned for information only. The program also scales the input and output for weapon yields other than one kiloton.

Summary of the Curve Fitting Process

This section was included to quickly summarize the major steps used to build the polynomial equations. It is intended as an outline for someone to follow if attempting a similar project.

1. A data set was extracted from the constant peak overpressure contour graphs (Figures 1 through 9).

2. The data points were then individually validated against the overpressure function to insure agreement. When working in the low peak overpressure region, at least two decimal places were required for peak overpressure values in order to extract an accurate ground range.

3. The tested data was entered into the computer as a separate data file.

4. A fourth or fifth degree polynomial was built using a backward elimination process. The process started with all possible combinations of variables and then removed those variables that did not contribute significantly to the solution. As each variable was removed, the equation was tested for best fit using minimum MEAN SQUARE ERROR as a

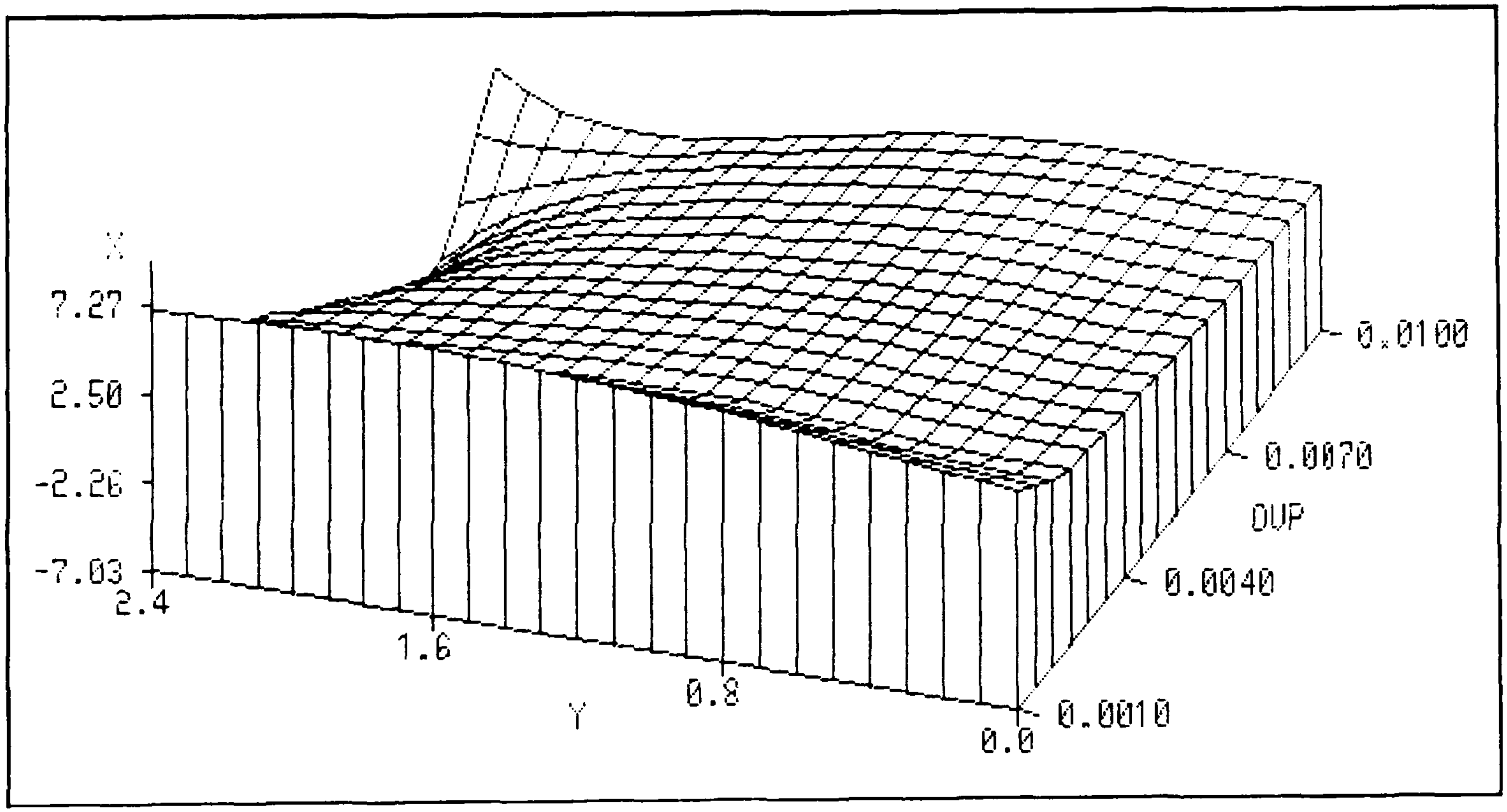

Figure 19. Three dimensional plot of Region 5 data showing X going positive (left rear quadrant) outside the range of actual data.

guide. The SAS PROC REG procedure was used to accomplish the fit.

5. The resulting polynomial was entered into a Fortran 77 program for testing. The program generated a set of predicted data for examination and graphics use.

6. SAS graphics were used to plot contours of the predicted and actual data for analysis and comparison.

IV. Results

Error Analysis

The nature of the data produced by Brode's overpressure function is so varied (wobbles in the contours, etc.) that it is difficult to present a concise description of the error in the new approximations for ground range. Curve fitting often rounds off sharp peaks or smooths out deep valleys in the original data. These peaks and valleys occur throughout the data produced by the overpressure function and are often limited to localized areas. To closely examine all these localized sources of error would require large quantities of statistical data. However, this requirement is not considered necessary to adequately cover the error present in the five ground range functions.

In order to present a meaningful representation of the true accuracy of the new approximations for ground range, the data was divided into segments. These segments can best be explained by returning to Figures 1 through 9. In these figures, the constant peak overpressure contours have similar characteristics among each other. The contours, though variable, resemble a knee at times and are often referred to as knee curves (12:106). The characteristic knee was considered an ideal reference point for the analysis of the error of the ground range function.

There were various reasons for selecting this reference point. The fit for the data ranging from the surface (Y = 0) to the knee was usually better than beyond the knee, where the values of ground range decreased rapidly with increasing burst height and were subsequently more difficult to fit. AF/SASM considered the region from the surface to and including the knee to be most important to their needs (7). The greatest errors often occurred where ground range (X) became zero. A percent error based on ground range was meaningless under these circumstances.

Consequently, the approximations' error analysis is presented in three segments. These segments are depicted in Figure 20 for clarity. The first segment covers the data from the surface to the knee. For this segment, the following three pieces of information are provided for each peak overpressure region: the largest error in feet, the largest error as a percentage of ground range, and the average error in feet. Similar figures are provided for data above the knee. The percentage figure was left out because, as mentioned previously, the value is meaningless. The third error analysis segment encompasses the entire knee. It provides maximum error values and percentages from the surface to a point beyond the knee that equals the value of ground range at the surface. This segment can best be visualized by examining Figure 20. The contours behave similarly in all regions except Region 3 where the knee does

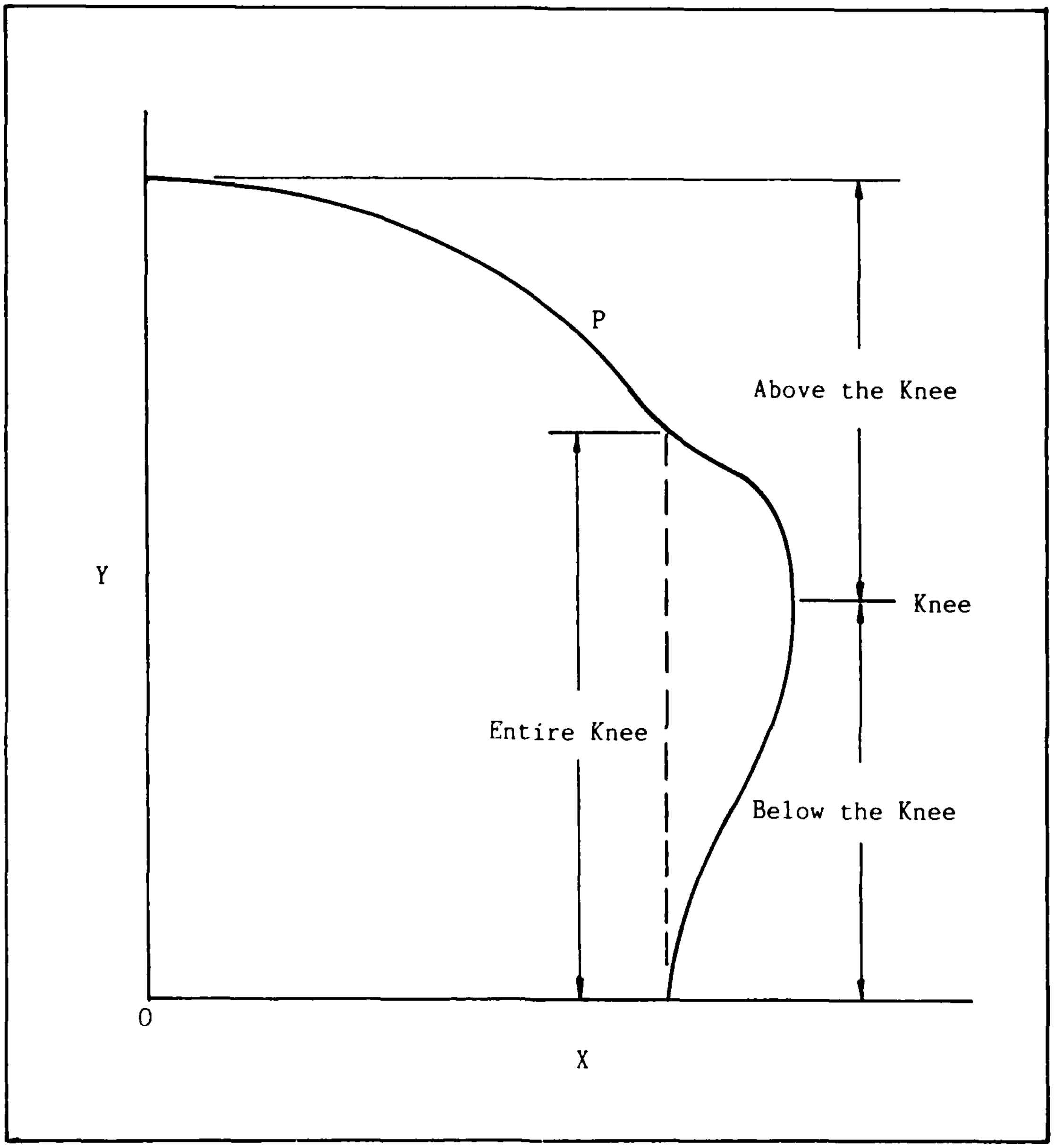

Figure 20. Division of a sample knee curve for error analysis purposes.

not protrude sufficiently to cross the surface ground range value a second time. The above error analysis is presented in Table 4.

Additionally, Table 4 provides statistical data from the curve fitting process. Most of the entries are self explanatory. The ADJ R^2 is a statistical measure of the goodness of the fit. The Range of X at $Y = 0$ provides a reference of the major ground range values covered by each region. For example, the peak overpressure values of 1000 to 10,000 psi in Region 2 primarily occur 71 to 154 feet from ground zero for a one kiloton explosion.

Table 4 was set up for someone to assess the acceptability of the error in the approximation for each region. The error terms are the maximum encountered for each segment and over the entire fit. They were extracted from an examination of the residuals. Average errors were also included to show that the largest error terms were both infrequent and did not reflect the fit as a whole. The average errors were computed by averaging the residuals of the high and low peak overpressures for each region. This decision was based on examination of residual versus peak overpressure plots (Figure 16 for example). For each region, lowest error occurred at high peak overpressures and highest error at low peak overpressures. The average of the two provided a reasonable approximation of the average error. Below the knee, the approximations were all within

Table 4. Summary of Statistical Information by Region.

	Region1	Region2	Region3	Region4	Region5
Peak overpressure range (ksi):	10-100	1-10	.1-1	.01-.1	.001-.01
Number of data points used:	147	123	133	138	169
Parameters estimated:	17	17	20	23	30
Type of polynomial (degree):	4	4	4	5	5
ADJ R^2 of fit:	.9841	.9838	.9892	.9877	.9957
Standard deviation of fit (ft):	2.19	4.97	8.89	35.56	113.3
Range of X at Y=0 (ft):	33-71	71-154	154-360	360-1030	1030-4680
Below the Knee					
Largest error (ft):	3.46	3.63	10.4	37.7	285.3
Largest error (percent of X):	7.8%	3.1%	3.7%	4.5%	5.5%
Average error (ft):	1.0	1.4	2.7	8.5	82.9
Entire Knee					
Largest approximate error (ft):	4.5	4.8	10.4	88.4	371.8
Largest error (percent of X):	10.9%	5.9%	3.7%	7.4%	11.5%
Above the Knee					
Largest error (ft):	6.7	15.1	31.5	109.9	371.8
Average error (ft):	2.6	4.6	9.2	41.4	122.3

the initial error tolerance of ten percent of the ground range. For the entire knee, Regions 1 and 5 had error terms larger than ten percent. These error terms were the largest for that segment and occurred only in isolated locations.

Polynomial Approximations

The information presented in Table 4 demonstrates that the new approximations to find scaled ground range perform this task adequately. Each approximation and its corresponding constant ground range contour plot are provided in Figures 21 through 25. Figures 27 through 31 in Appendix C provide a comparison of predicted data to actual data. These figures visually demonstrate the closeness of the overall fit.

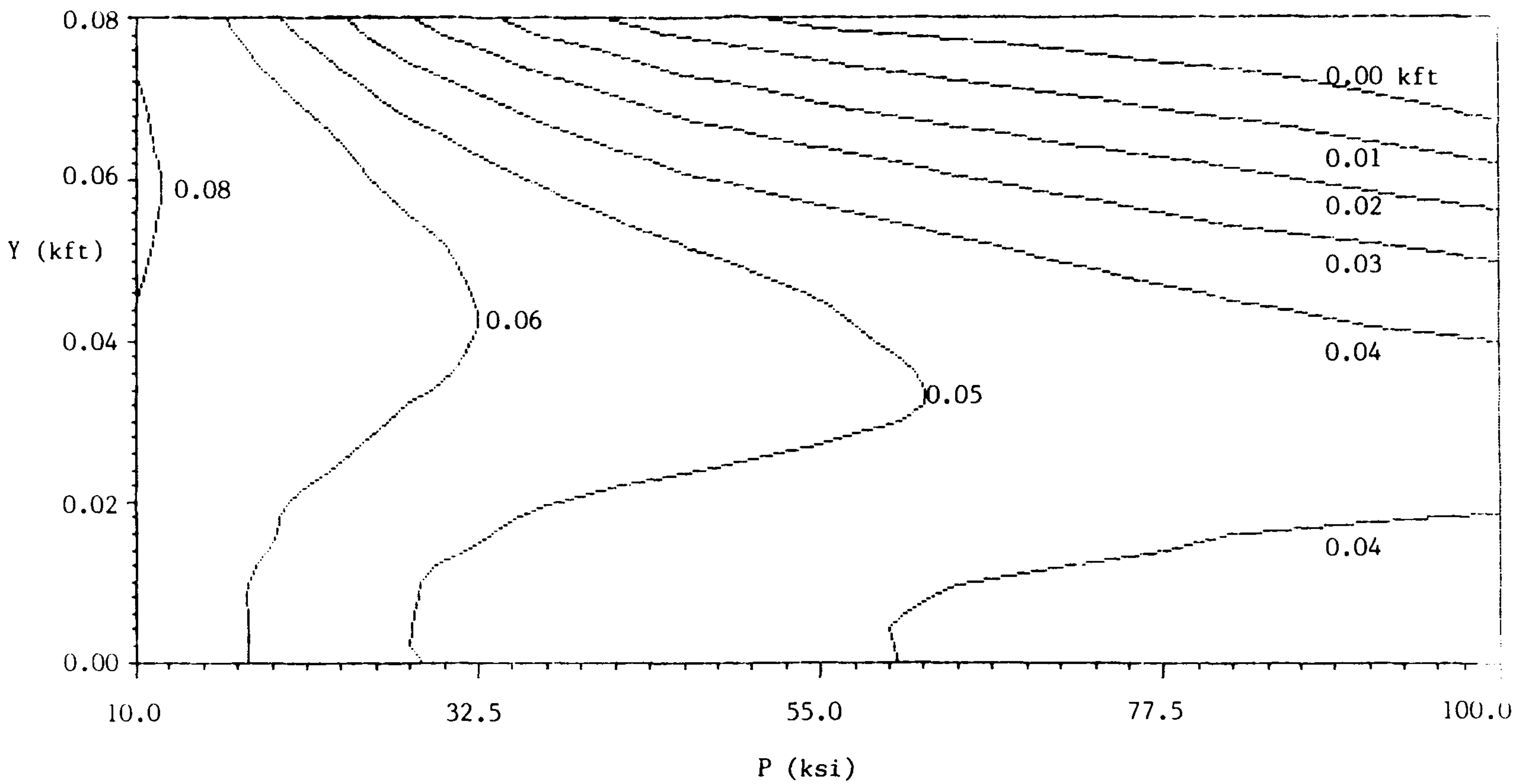

Figure 21. Constant scaled ground range contours for Region 1 generated by:

$$X = .09125435 - .00259593P - .011649PY + .0000538051P^2 + .0001111895P^2Y + 1.55066730Y^2P - .0156667P^2Y^2 - 5.23760\times10^{-7}P^3 + .00004916175P^3Y^2 + 97.31778176Y^3 - 25.706Y^3P + 1.86726\times10^{-9}P^4 - 776.975Y^4 + 80.37406298Y^4P + .10802327Y^3P^2 + .01220085Y^4P^3 - .000106692P^4Y^4$$

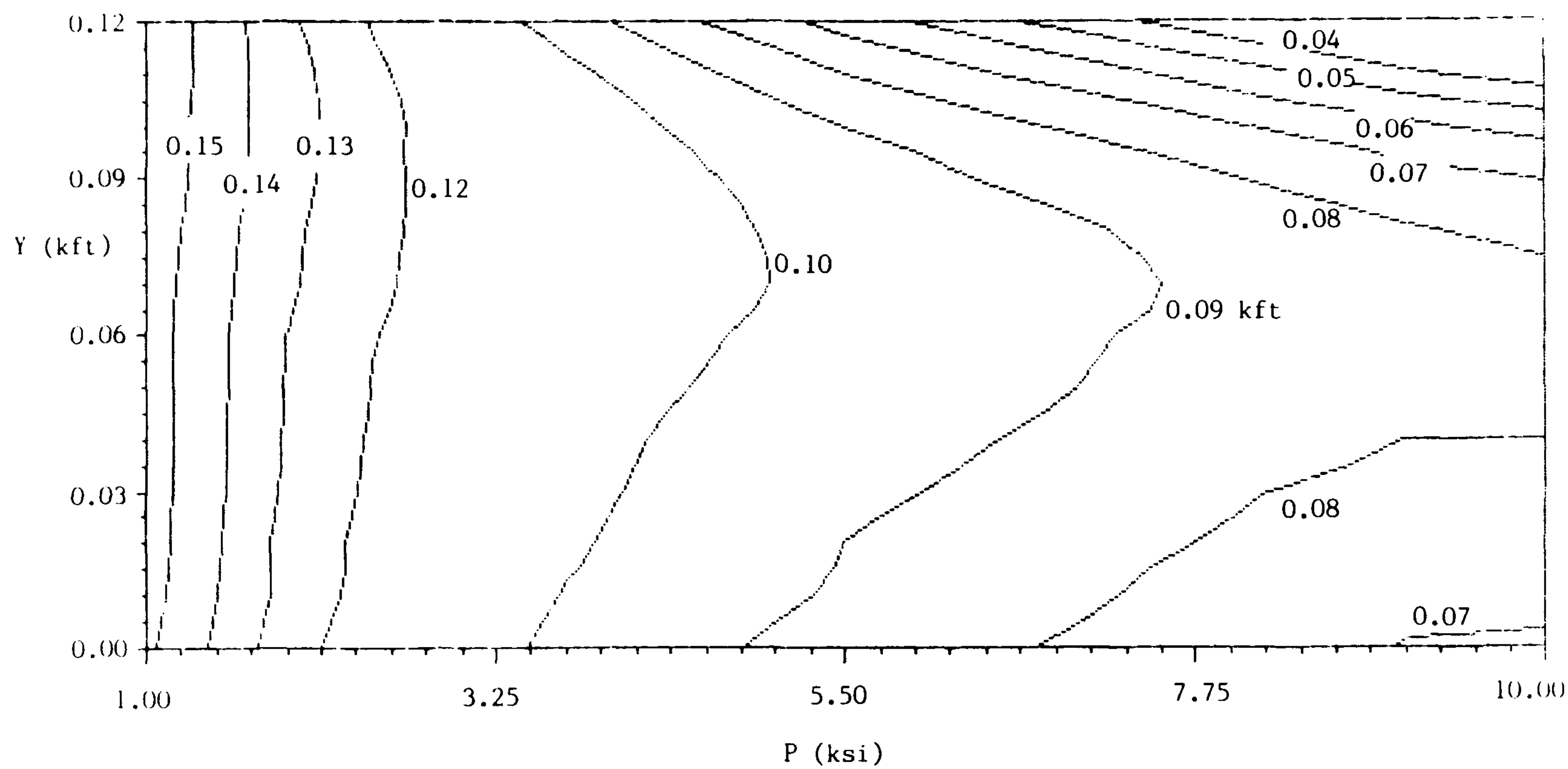

Figure 22. Constant scaled ground range contours for Region 2 generated by:

$$X = .20154762 - .0621137P + .26548571Y + .01362242P^2 - 7.29235Y^2 + .07036665P^2Y^2 - .00141394P^3 + 39.26591463Y^3 + 47.69269458Y^3P + 1.41551464P^3Y^3 + .00005402714P^4 - .0671128P^4Y^3 - 314.529Y^4P - 12.1885Y^3P^2 + 64.01100943Y^4P^2 - 8.28738Y^4P^3 + .42330598P^4Y^4$$

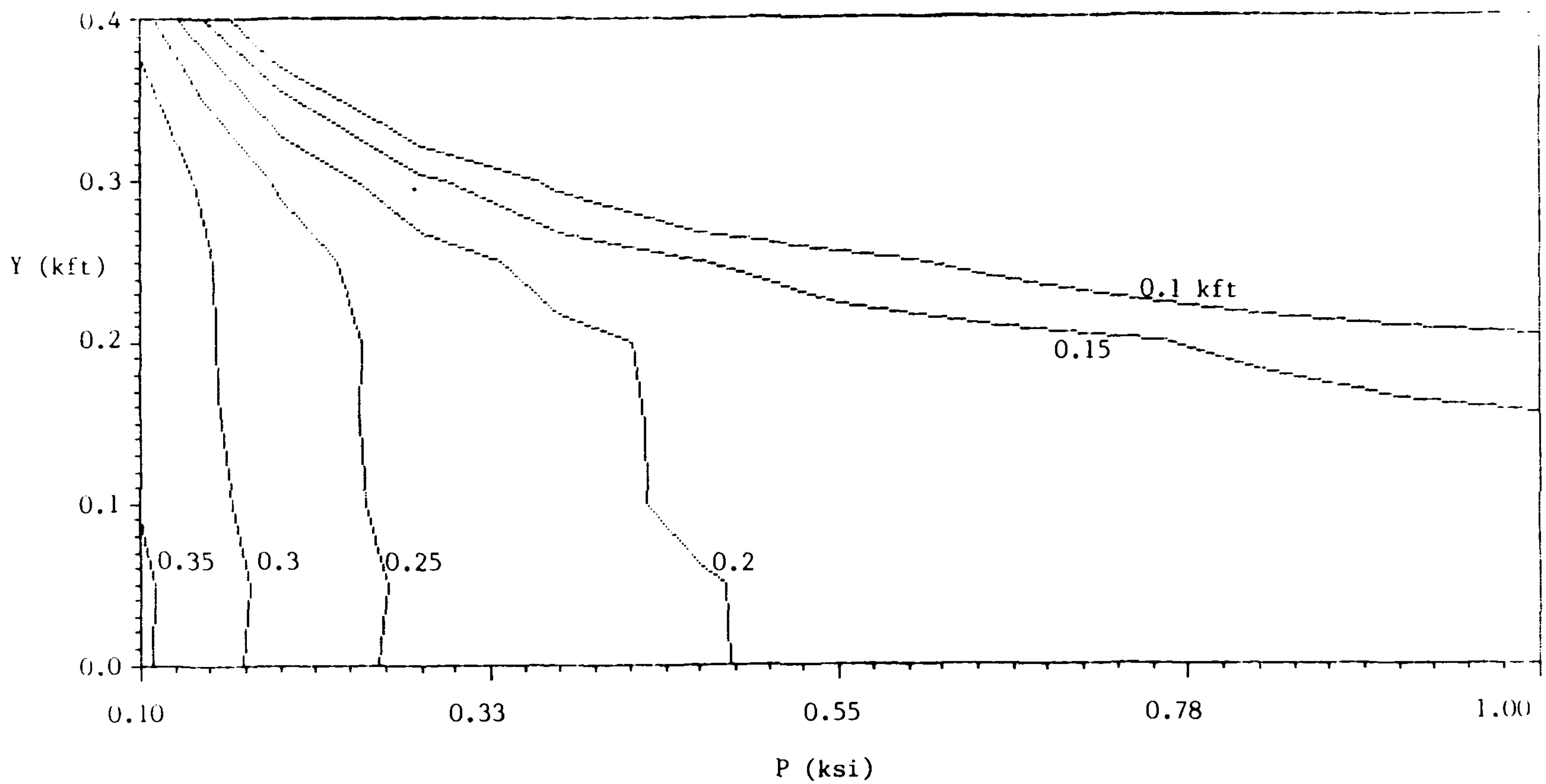

Figure 23. Constant scaled ground range contours for Region 3 generated by:

$$X = .49576495 - 1.76963P + .2873705Y + 4.11651645P^2 - 7.91446Y^2 + 30.88152847Y^2P - 157.536P^2Y^2 - 4.38405P^3 + 237.78055884P^3Y^2 + 30.88701918Y^3 - 90.0484Y^3P - 1383.45P^3Y^3 + 1.69350254P^4 - 109.732P^4Y^2 + 631.54888112P^4Y^3 - 29.9186Y^4 + 876.59060473Y^3P^2 - 1444.15Y^4P^2 + 2183.11803Y^4P^3 - 935.61P^4Y^4$$

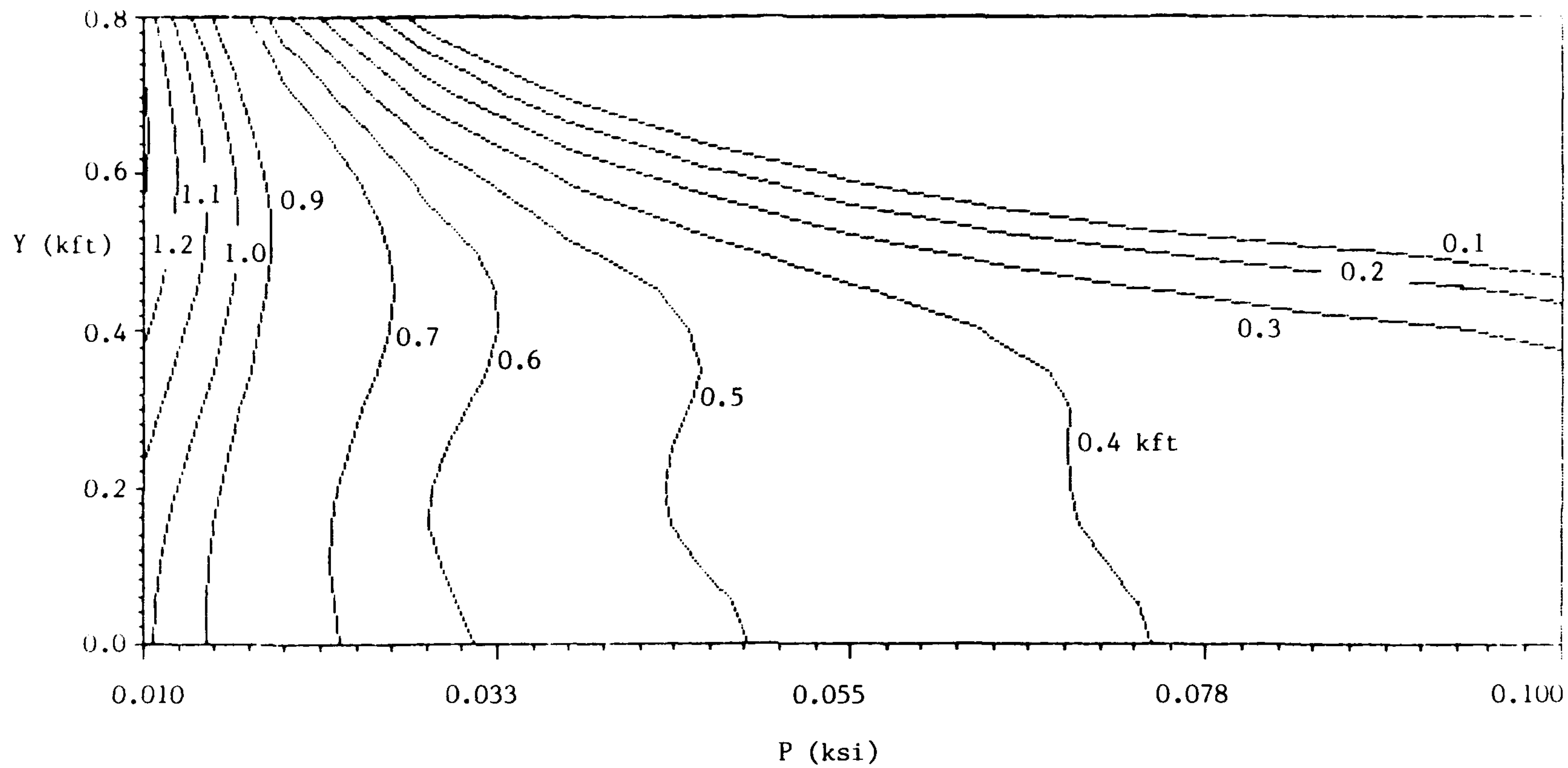

Figure 24. Constant scaled ground range contours for Region 4 generated by:

$$X = 1.62275752 - 83.2005P + 2705.11147P^2 + 6.12568509Y^2 - 546.803Y^2P + 10261.68969P^2Y^2 - 45666.2P^3 - 56907.7P^3Y^2 - 14.3825Y^3 + 1654.94706Y^3P + 375478.28531P^4 + 15.10099190Y^4 - 1985.89Y^4P - 22491Y^3P^2 + 18051.29615Y^4P^2 + 4798899.81P^4Y^4 - 1186924P^5 + 8777484.91P^5Y^3 - 48910776P^5Y^4 - 5.23814Y^5 + 660.83710834Y^5P - 5735942Y^5P^4 + 49158205.90P^5Y^5$$

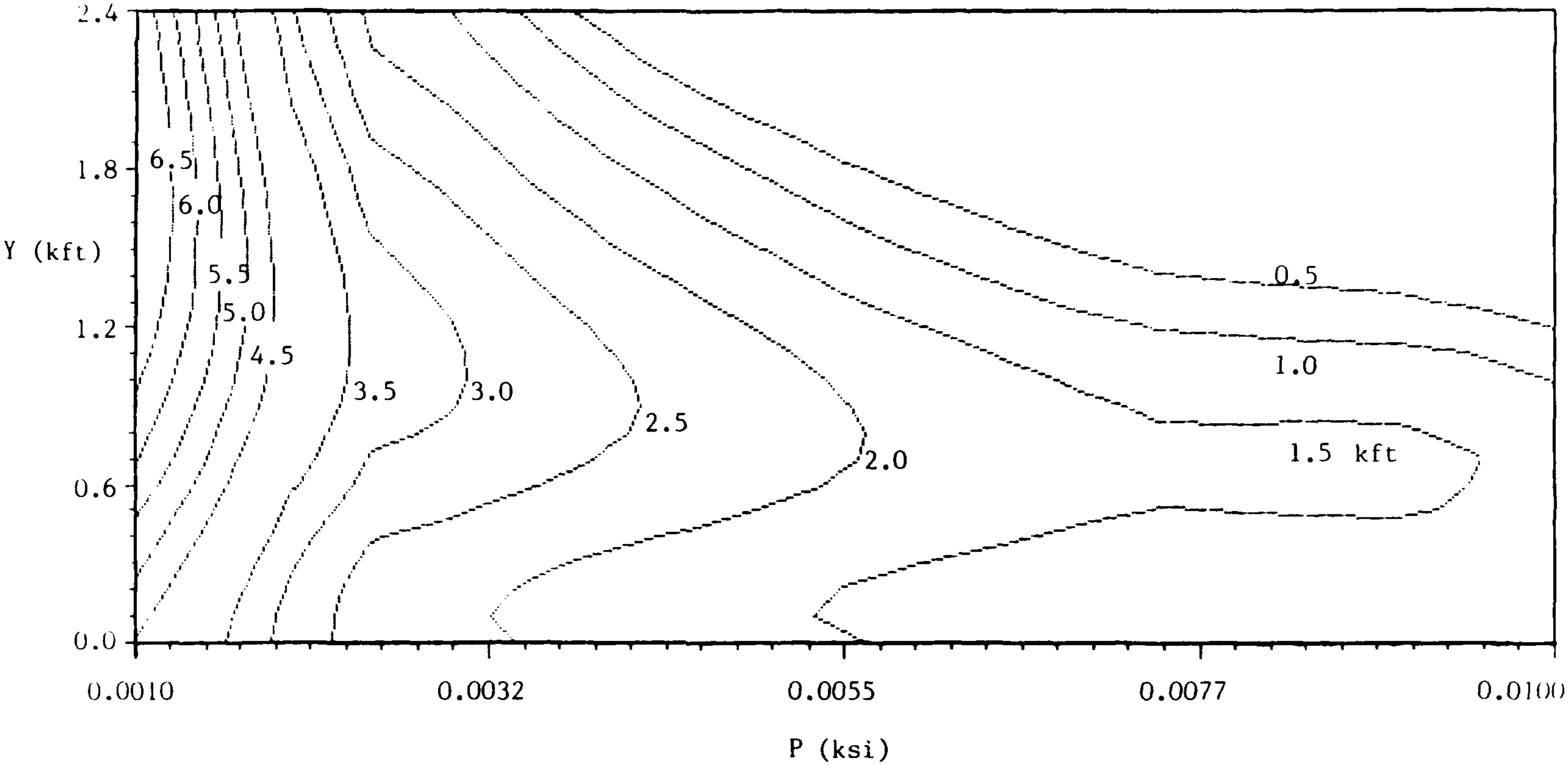

Figure 25. Constant scaled ground range contours for Region 5 generated by:

$$X = 8.23926119 - 5306.71P + 6.05714948Y - 5471.77PY + 1828923.98P^2 + 1686307.36P^2Y - 2.89962Y^2 + 4167.17167Y^2P - 352120P^2Y^2 - 31.9297925\times10^7P^3 - 27.9705884\times10^7P^3Y + .87118870Y^3 - 1187.34Y^3P + 162.39402197\times10^6P^3Y^2 + 269.81456091\times10^8P^4 + 240.17907175\times10^8P^4Y - 103.60283247\times10^8P^4Y^3 - 219.221Y^4P - 855214Y^3P^2 + 501833.53514Y^4P^2 - 47.312304\times10^6Y^4P^3 - 8.75407\times10^{11}P^5 - 8.08007\times10^{11}P^5Y + 3361.23611805\times10^8P^5Y^3 - .0104459Y^5 + 34.21894224Y^5P - 31509.7Y^5P^2 - 11250907Y^5P^3 + 203.1223752\times10^7Y^5P^4 - 6059.3995968\times10^7P^5Y^5$$

V. Conclusions and Recommendations

Conclusions

This thesis has demonstrated a method of finding the scaled ground range directly from values of peak overpressure and scaled burst height. Five polynomial equations were developed through curve fitting approximation techniques to cover data for 1 to 100,000 psi peak overpressures. The five polynomial equations were combined into a Fortran 77 computer program (Appendix B). The program, though built to stand alone, can easily be modified into a subroutine and incorporated into a larger computer program. Since the polynomial equations were based on scaled data for one kiloton, the program also scales the input and output for other weapon yields.

The Brode expression was used to evaluate the accuracy of the five polynomial equations. The error was computed as a percentile for high ground range values and extracted as a quantitative error for both low and high ground range values. In most cases, the error of the new approximation was well below ten percent of the actual ground range for high ground range values. There were two instances where the error was 10.9 and 11.5 percent of the ground range. These two cases were isolated and not indicative of the overall fit. Some larger errors, however, were encountered with the lower ground ranges (Table 4). The user of these

approximations must personally decide whether these errors are acceptable.

Recommendations

The curve fitting methodology presented in this thesis should not be considered the only way of solving this type of problem. It is just one example. For instance, the peak overpressure values were divided into five regions because of their exponential nature, and each region was solved separately. It is conceivable to assume that the peak overpressure could be linearized by using logarithms to transform the exponential range of peak overpressure to a linear range. Then a fit of larger regions or possibly the entire region could be attempted. Of course, the data would also have to be transformed, but the method is feasible.

The problem was originally partitioned to make it more manageable and easier to work with. However, there are other ways to partition this problem that might be more advantageous to error reduction. Two possible approaches are depicted in Figure 26.

Figure 26a portrays peak overpressure contours like those in Figure 1. The functional relationship $X = f(P,Y)$ is not possible here because there are several values of X for the same combination of P and Y. To accurately fit this data, partitioning like that depicted in Figure 26a would probably be required. Even then, the solution might still be indeterminate.

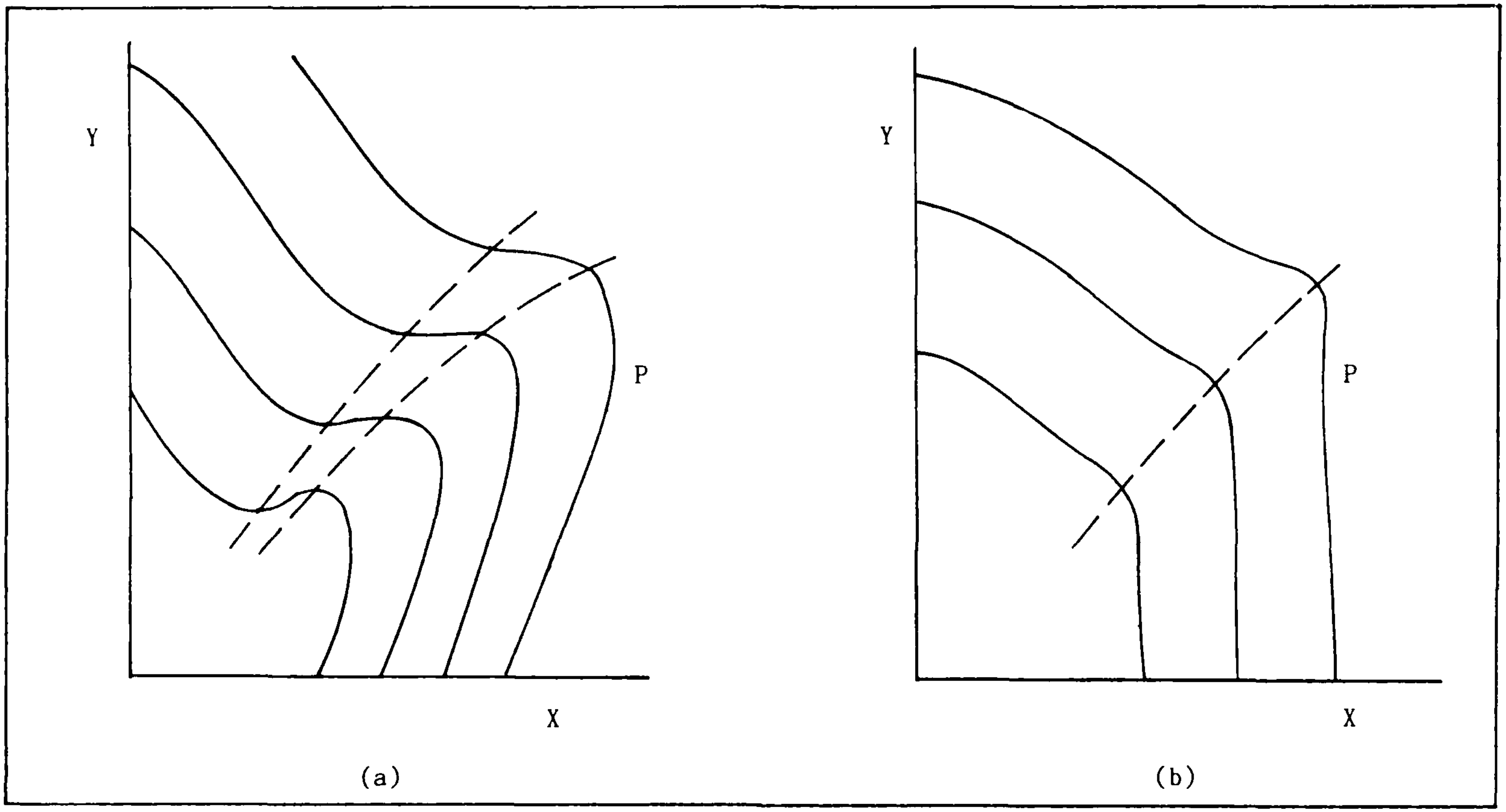

Figure 26. Alternative ways to partition the data.

Figure 26b represents an alternative partitioning scheme to that used in this thesis. The partition divides a contour into two curves with similar characteristics. The data above and below the partition could then be fit with a lower degree polynomial. Also, the similarities of the curves on each side of the partition would probably contribute to an overall error reduction over the entire surface.

Additional data was added to those regions of the fit that had the largest errors in order to give them a greater weight during the least squares fitting process. Since no error reduction occurred, the amount of data added may have been insufficient to adequately weight these regions. The use of more data in this regard may still be an effective means of reducing the error.

Further refinement of the current polynomial equations is also possible. Sixth degree polynomials were only lightly examined as contributors to the fit. The disadvantages of using higher degree polynomials have already been discussed in this thesis.

Although not used, a bisection routine would have been helpful to generate data sets, especially uniformly spaced data. Peak overpressure data was not uniformly spaced during the research, and this condition occasionally caused the curves to wander.

The methodology presented by this thesis might also be applicable to other nuclear effects data. For example, computing the ground range for peak dynamic pressure, though possibly more difficult to do, has similar characteristics to the problem presented in this thesis.

The overpressure function is not in its final form. The high overpressure region was undergoing revision while this thesis was being written (3). If the revision only changes a portion of the data, some of the polynomial equations may not require updating. A complete change of the data will require new approximations for scaled ground range. Whatever the outcome, this thesis remains as a guide for that purpose.

Appendix A: Data

This section contains the data sets used to obtain polynomial approximations for Regions 1 through 5. The data was originally extracted from Figures 1 through 9, then validated against the overpressure function for accuracy.

The validation process involved interpolating the scaled ground range (X) given values of peak overpressure (P) and scaled burst height (Y). The accuracies of the interpolated data is provided in Table 5.

Table 5. Accuracy of interpolated scaled ground range data.

Region	Accuracy of X (kft)
1	.0001 - .0002
2	.0001 - .0005
3	.0001 - .0005
4	.0005 - .005
5	.001 - .01

In most cases, the data extracted from Figures 1 through 9 closely matched the data generated by the overpressure function. However, in Region 5, some variation was found between corresponding graphical versus analytical data values. Where the discrepancy was large, the graphical data was selected because it was consistent with the majority of the data. These data points are identified in the Region 5 data listing of this appendix.

This data set was used to build the polynomial for Region 1.

X(kft)	Y(kft)	P(psi)	X	Y	P
.033	0.00	100000	.0465	0.02	50000
.0337	0.005	100000	.0487	0.025	50000
.0361	0.01	100000	.0508	0.03	50000
.0374	0.015	100000	.0525	0.035	50000
.0395	0.02	100000	.0537	0.04	50000
.0417	0.025	100000	.0542	0.045	50000
.0435	0.03	100000	.0536	0.048	50000
.0446	0.035	100000	.0468	0.05	50000
.0447	0.038	100000	.0407	0.055	50000
.036	0.041	100000	.0369	0.06	50000
.0322	0.045	100000	.0323	0.065	50000
.0291	0.05	100000	.0184	0.073	50000
.0243	0.055	100000	.0109	0.075	50000
.0155	0.06	100000	.0447	0.00	40000
.008	0.062	100000	.045	0.005	40000
.0342	0.00	90000	.0468	0.01	40000
.0348	0.005	90000	.0482	0.015	40000
.0372	0.01	90000	.0492	0.02	40000
.0384	0.015	90000	.0512	0.025	40000
.0405	0.02	90000	.0534	0.03	40000
.0427	0.025	90000	.0553	0.035	40000
.0445	0.03	90000	.0567	0.04	40000
.0458	0.035	90000	.0575	0.045	40000
.0458	0.04	90000	.0575	0.05	40000
.0346	0.045	90000	.0564	0.052	40000
.0312	0.05	90000	.0525	0.053	40000
.0271	0.055	90000	.0479	0.055	40000
.020	0.06	90000	.0425	0.06	40000
.0372	0.00	70000	.0386	0.065	40000
.0377	0.005	70000	.0335	0.07	40000
.0399	0.01	70000	.0259	0.075	40000
.0411	0.015	70000	.0102	0.08	40000
.0429	0.02	70000	.0491	0.00	30000
.0451	0.025	70000	.051	0.01	30000
.0471	0.03	70000	.0532	0.02	30000
.0486	0.035	70000	.0571	0.03	30000
.0494	0.04	70000	.0607	0.04	30000
.0491	0.043	70000	.0624	0.05	30000
.042	0.045	70000	.0619	0.055	30000
.0364	0.05	70000	.0556	0.058	30000
.0329	0.055	70000	.0523	0.06	30000
.0283	0.06	70000	.0424	0.07	30000
.023	0.065	70000	.0304	0.08	30000
.0114	0.068	70000	.0186	0.085	30000
.0415	0.00	50000	.0088	0.087	30000
.0419	0.005	50000	.0522	0.00	25000
.0439	0.01	50000	.0539	0.01	25000
.0452	0.015	50000	.056	0.02	25000

X(kft)	Y(kft)	P(psi)
.0596	0.03	25000
.0633	0.04	25000
.0654	0.05	25000
.0656	0.055	25000
.065	0.058	25000
.0633	0.06	25000
.0478	0.07	25000
.0383	0.08	25000
.0307	0.085	25000
.0177	0.09	25000
.0041	0.092	25000
.0562	0.00	20000
.0577	0.01	20000
.0598	0.02	20000
.0628	0.03	20000
.0668	0.04	20000
.0692	0.05	20000
.0697	0.055	20000
.0695	0.06	20000
.0639	0.065	20000
.0562	0.07	20000
.0465	0.08	20000
.0337	0.09	20000
.0218	0.095	20000
.0133	0.097	20000
.0618	0.00	15000
.063	0.01	15000
.0652	0.02	15000
.0675	0.03	15000
.0714	0.04	15000
.0743	0.05	15000
.0754	0.06	15000
.075	0.065	15000
.0713	0.07	15000
.0565	0.08	15000
.0473	0.09	15000
.0319	0.10	15000
.0158	0.105	15000
.0705	0.00	10000
.0715	0.01	10000
.0735	0.02	10000
.0755	0.03	10000
.079	0.04	10000
.082	0.05	10000
.0838	0.06	10000
.084	0.07	10000
.077	0.08	10000
.0635	0.09	10000
.0545	0.10	10000

X	Y	P
.0405	0.11	10000
.01	0.119	10000

This data set was used to build the polynomial for Region 2.

X(kft)	Y(kft)	P(psi)	X	Y	P	X	Y	P
.0705	0.00	10000	.100	0.06	5000	.134	0.00	1500
.0715	0.01	10000	.1012	0.07	5000	.134	0.02	1500
.0735	0.02	10000	.1013	0.08	5000	.137	0.04	1500
.0755	0.03	10000	.1005	0.09	5000	.1376	0.06	1500
.079	0.04	10000	.0925	0.10	5000	.1405	0.08	1500
.082	0.05	10000	.079	0.11	5000	.1416	0.10	1500
.0838	0.06	10000	.0695	0.12	5000	.1406	0.12	1500
.084	0.07	10000	.064	0.125	5000	.137	0.14	1500
.077	0.08	10000	.0575	0.13	5000	.1133	0.16	1500
.0635	0.09	10000	.046	0.135	5000	.0913	0.18	1500
.0545	0.10	10000	.0375	0.14	5000	.0525	0.20	1500
.0405	0.11	10000	.0013	0.146	5000	.002	0.208	1500
.001	0.12	10000	.096	0.00	4000	.154	0.00	1000
.076	0.00	8000	.0975	0.02	4000	.154	0.02	1000
.077	0.01	8000	.1005	0.04	4000	.156	0.04	1000
.0787	0.02	8000	.1057	0.06	4000	.156	0.06	1000
.0805	0.03	8000	.1077	0.08	4000	.1574	0.08	1000
.0833	0.04	8000	.1055	0.10	4000	.159	0.10	1000
.0865	0.05	8000	.0823	0.12	4000	.1586	0.12	1000
.0885	0.06	8000	.0725	0.13	4000	.1577	0.14	1000
.0892	0.07	8000	.0595	0.14	4000	.1545	0.16	1000
.088	0.08	8000	.038	0.15	4000	.131	0.18	1000
.0765	0.09	8000	.0013	0.156	4000	.108	0.20	1000
.0646	0.10	8000	.106	0.00	3000	.094	0.21	1000
.0545	0.11	8000	.107	0.02	3000	.075	0.22	1000
.038	0.12	8000	.1096	0.04	3000	.045	0.23	1000
.0011	0.127	8000	.1138	0.06	3000	.002	0.235	1000
.084	0.00	6000	.1165	0.08	3000			
.0845	0.01	6000	.116	0.10	3000			
.086	0.02	6000	.105	0.12	3000			
.0876	0.03	6000	.081	0.14	3000			
.0898	0.04	6000	.0685	0.15	3000			
.093	0.05	6000	.051	0.16	3000			
.0954	0.06	6000	.036	0.165	3000			
.0964	0.07	6000	.0015	0.17	3000			
.0963	0.08	6000	.1215	0.00	2000			
.0935	0.09	6000	.122	0.02	2000			
.081	0.10	6000	.125	0.04	2000			
.0695	0.11	6000	.127	0.06	2000			
.059	0.12	6000	.130	0.08	2000			
.0425	0.13	6000	.1303	0.10	2000			
.0012	0.1385	6000	.129	0.12	2000			
.089	0.00	5000	.1134	0.14	2000			
.0896	0.01	5000	.09	0.16	2000			
.091	0.02	5000	.077	0.17	2000			
.0927	0.03	5000	.058	0.18	2000			
.0943	0.04	5000	.044	0.185	2000			
.0973	0.05	5000	.0016	0.191	2000			

This data set was used to build the polynomial for Region 3.

X(kft)	Y(kft)	P(psi)	X	Y	P	X	Y	P
.154	0.00	1000	.1725	0.18	700	.005	0.387	200
.154	0.01	1000	.149	0.20	700	.307	0.00	150
.154	0.02	1000	.125	0.22	700	.305	0.04	150
.1545	0.03	1000	.0115	0.23	700	.3035	0.08	150
.156	0.04	1000	.095	0.24	700	.2955	0.12	150
.1565	0.05	1000	.0715	0.25	700	.2935	0.16	150
.156	0.06	1000	.005	0.262	700	.29	0.20	150
.1565	0.07	1000	.1965	0.00	500	.286	0.24	150
.1574	0.08	1000	.197	0.04	500	.29	0.28	150
.1585	0.09	1000	.195	0.08	500	.288	0.30	150
.159	0.10	1000	.196	0.12	500	.274	0.32	150
.159	0.11	1000	.1935	0.16	500	.235	0.34	150
.1586	0.12	1000	.192	0.20	500	.21	0.36	150
.158	0.13	1000	.171	0.22	500	.179	0.38	150
.1577	0.14	1000	.1445	0.24	500	.138	0.40	150
.1575	0.15	1000	.132	0.25	500	.1095	0.41	150
.1545	0.16	1000	.117	0.26	500	.068	0.42	150
.148	0.17	1000	.098	0.27	500	.005	0.426	150
.131	0.18	1000	.071	0.28	500	.36	0.00	100
.119	0.19	1000	.005	0.29	500	.358	0.04	100
.108	0.20	1000	.2365	0.00	300	.355	0.08	100
.094	0.21	1000	.2355	0.04	300	.35	0.12	100
.075	0.22	1000	.234	0.08	300	.343	0.16	100
.045	0.23	1000	.231	0.12	300	.3415	0.20	100
.002	0.235	1000	.2295	0.16	300	.338	0.24	100
.163	0.00	850	.2265	0.20	300	.334	0.28	100
.1625	0.02	850	.2285	0.22	300	.3375	0.32	100
.1645	0.04	850	.226	0.24	300	.3365	0.34	100
.1645	0.06	850	.2025	0.26	300	.327	0.36	100
.165	0.08	850	.1735	0.28	300	.292	0.38	100
.167	0.10	850	.147	0.30	300	.2615	0.40	100
.1665	0.12	850	.1075	0.32	300	.234	0.42	100
.1655	0.14	850	.078	0.33	300	.201	0.44	100
.165	0.16	850	.005	0.34	300	.157	0.46	100
.154	0.18	850	.275	0.00	200	.128	0.47	100
.1255	0.20	850	.273	0.04	200	.088	0.48	100
.100	0.22	850	.272	0.08	200	.005	0.488	100
.081	0.23	850	.265	0.12	200			
.0535	0.24	850	.264	0.16	200			
.1745	0.00	700	.26	0.20	200			
.174	0.02	700	.2605	0.24	200			
.1755	0.04	700	.262	0.26	200			
.176	0.06	700	.257	0.28	200			
.175	0.08	700	.227	0.30	200			
.1765	0.10	700	.199	0.32	200			
.177	0.12	700	.171	0.34	200			
.1755	0.14	700	.134	0.36	200			
.175	0.16	700	.072	0.38	200			

This data set was used to build the polynomial for Region 4.

X(kft)	Y(kft)	P(psi)	X	Y	P	X	Y	P
.36	0.00	100	.4795	0.00	50	.523	0.75	20
.358	0.04	100	.479	0.05	50	.457	0.80	20
.355	0.08	100	.4745	0.10	50	.355	0.85	20
.35	0.12	100	.471	0.15	50	.196	0.90	20
.343	0.16	100	.463	0.20	50	.140	0.91	20
.3415	0.20	100	.463	0.25	50	.008	0.92	20
.338	0.24	100	.464	0.30	50	.836	0.00	15
.334	0.28	100	.465	0.35	50	.850	0.10	15
.3375	0.32	100	.4655	0.40	50	.870	0.20	15
.3365	0.34	100	.4625	0.43	50	.90	0.30	15
.327	0.36	100	.442	0.46	50	.94	0.40	15
.292	0.38	100	.382	0.49	50	.984	0.50	15
.2615	0.40	100	.337	0.52	50	1.016	0.60	15
.234	0.42	100	.292	0.55	50	1.01	0.70	15
.201	0.44	100	.233	0.58	50	.955	0.75	15
.157	0.46	100	.180	0.60	50	.77	0.80	15
.128	0.47	100	.145	0.61	50	.615	0.85	15
.088	0.48	100	.096	0.62	50	.56	0.90	15
.005	0.488	100	.007	0.627	50	.47	0.95	15
.384	0.00	85	.601	0.00	30	.35	1.00	15
.382	0.05	85	.603	0.05	30	.13	1.05	15
.378	0.10	85	.601	0.10	30	.01	1.06	15
.3685	0.15	85	.602	0.15	30	1.03	0.00	10
.365	0.20	85	.602	0.20	30	1.055	0.10	10
.362	0.25	85	.600	0.25	30	1.09	0.20	10
.358	0.30	85	.607	0.30	30	1.14	0.30	10
.3605	0.35	85	.616	0.35	30	1.195	0.40	10
.317	0.40	85	.627	0.40	30	1.26	0.50	10
.256	0.44	85	.637	0.45	30	1.31	0.60	10
.184	0.48	85	.641	0.50	30	1.34	0.70	10
.082	0.51	85	.633	0.54	30	1.33	0.80	10
.416	0.00	70	.550	0.58	30	1.29	0.85	10
.414	0.05	70	.442	0.62	30	1.18	0.90	10
.410	0.10	70	.390	0.66	30	.98	0.95	10
.402	0.15	70	.316	0.70	30	.89	1.00	10
.397	0.20	70	.212	0.74	30	.76	1.05	10
.395	0.25	70	.175	0.75	30	.72	1.10	10
.392	0.30	70	.125	0.76	30	.65	1.15	10
.392	0.35	70	.007	0.77	30	.55	1.20	10
.384	0.40	70	.726	0.00	20	.41	1.25	10
.340	0.43	70	.735	0.10	20	.20	1.30	10
.2925	0.46	70	.746	0.20	20	.02	1.315	10
.247	0.49	70	.762	0.30	20			
.1845	0.52	70	.792	0.40	20			
.156	0.53	70	.828	0.50	20			
.121	0.54	70	.843	0.60	20			
.065	0.55	70	.820	0.65	20			
.005	0.554	70	.660	0.70	20			

This data set was used to build the polynomial for Region 5. Values of X marked with a (*) were extracted directly from Figures 8 and 9 because they varied significantly to those values generated by the overpressure function.

X(kft)	Y(kft)	P(psi)	X	Y	P	X	Y	P
1.03	0.00	10	1.699	0.50	6	1.85	2.20	3
1.057	0.10	10	1.788	0.60	6	1.54	2.40	3
1.089	0.20	10	1.86	0.70	6	1.20*	2.60	3
1.138	0.30	10	1.902	0.80	6	.95*	2.70	3
1.196	0.40	10	1.909	0.85	6	.60*	2.80	3
1.256	0.50	10	1.906	0.90	6	2.8	0.00	2
1.309	0.60	10	1.89	0.95	6	2.99	0.20	2
1.328	0.65	10	1.86	1.00	6	3.27	0.40	2
1.34	0.70	10	1.728	1.10	6	3.55	0.60	2
1.34	0.75	10	1.344	1.20	6	3.78	0.80	2
1.327	0.80	10	1.253	1.30	6	3.99	1.00	2
1.29	0.85	10	1.095	1.40	6	4.17	1.20	2
1.188	0.90	10	.97	1.50	6	4.23	1.30	2
.985	0.95	10	.835	1.60	6	4.27	1.40	2
.897	1.00	10	.615	1.70	6	4.24	1.50	2
.765	1.05	10	.28	1.78	6	4.18	1.60	2
.718	1.10	10	1.75	0.00	4	3.9	1.80	2
.649	1.15	10	1.88	0.20	4	3.57	2.00	2
.552	1.20	10	2.05	0.40	4	3.25	2.20	2
.42	1.25	10	2.255	0.60	4	2.96	2.40	2
.205	1.30	10	2.457	0.80	4	2.77	2.60	2
.02	1.315	10	2.524	0.90	4	2.64	2.80	2
1.163	0.00	8	2.555	1.00	4	2.4	3.00	2
1.196	0.10	8	2.534	1.10	4	2.08	3.20	2
1.236	0.20	8	2.465	1.20	4	1.80*	3.40	2
1.292	0.30	8	2.12	1.40	4	1.35*	3.60	2
1.363	0.40	8	1.665	1.60	4	1.10*	3.70	2
1.437	0.50	8	1.52	1.80	4	.60*	3.80	2
1.503	0.60	8	1.225	2.00	4	3.46	0.00	1.5
1.549	0.70	8	1.105	2.10	4	3.65	0.20	1.5
1.560	0.75	8	.94	2.20	4	3.98	0.40	1.5
1.562	0.80	8	.40*	2.30	4	4.35	0.60	1.5
1.550	0.85	8	2.11	0.00	3	4.68	0.80	1.5
1.522	0.90	8	2.265	0.20	3	4.93	1.00	1.5
1.334	1.00	8	2.475	0.40	3	5.13	1.20	1.5
1.061	1.10	8	2.68	0.60	3	5.26	1.40	1.5
.865	1.20	8	2.905	0.80	3	5.33	1.60	1.5
.755	1.30	8	3.09	1.00	3	5.27	1.80	1.5
.554	1.40	8	3.14	1.10	3	4.97	2.00	1.5
.02	1.50	8	3.145	1.20	3	4.62	2.20	1.5
1.369	0.00	6	3.092	1.30	3	4.32	2.40	1.5
1.411	0.10	6	3.0	1.40	3	4.08	2.60	1.5
1.462	0.20	6	2.67	1.60	3	3.85	2.80	1.5
1.527	0.30	6	2.2	1.80	3	3.63	3.00	1.5
1.608	0.40	6	2.01	2.00	3	3.45	3.20	1.5

X(kft)	Y(kft)	P(psi)
3.30	3.40	1.5
3.1	3.60	1.5
2.8	3.80	1.5
2.5	4.00	1.5
2.15*	4.20	1.5
1.75*	4.40	1.5
1.10*	4.60	1.5
.50*	4.70	1.5
4.68	0.00	1
4.9	0.25	1
5.44	0.50	1
6.03	0.75	1
6.6	1.00	1
7.05	1.25	1
7.4	1.50	1
7.55	1.75	1
7.52	2.00	1
7.3	2.25	1
6.75	2.50	1
6.15	2.75	1
5.8	3.00	1
5.55	3.25	1
5.35	3.50	1
5.15	3.75	1
4.95	4.00	1
4.75	4.25	1
4.55	4.50	1
4.25	4.75	1
3.9	5.00	1
3.55	5.25	1
3.10*	5.50	1
2.00*	6.00	1
1.75*	6.10	1
1.10*	6.20	1

Appendix B: Computer Programs

This section contains the following representative computer programs:

1. Computerized version of the overpressure function which was used to generate data.
2. Sample SAS program to perform a regression analysis of a fifth degree polynomial.
3. Sample SAS/GRAPH program to generate response surface and contour graphics for a set of data.
4. Computerized version of the range function.

```
      PROGRAM OVERPR
      IMPLICIT NONE
C THIS PROGRAM COMPUTES AND PRINTS OUT THE SURFACE PEAK
C OVERPRESSURE IN PSI AS A FUNCTION OF WEAPON YIELD, RANGE
C FROM GROUND ZERO IN FEET, AND BURST HEIGHT IN FEET.
C IT IS BASED ON THE BRODE EQUATIONS.
C
C THESE ARE ALL THE VARIABLES IN THIS PROGRAM.
C THE IMPORTANT ONES ARE DEFINED BELOW.
C
      REAL R,X,Y,Z,AZ,BZ1,BZ2,BZ3,BZ,CZ1,CZ2,CZ,DZ1,DZ2,DZ
      REAL EZ,FZ1,FZ2,FZ,GZ,HZ1,HZ2,HZ3,HRY,HZRY,MY1,MY2,MY
      REAL NY,PS1,PS2,PS3,PS4,PS,GR,H,M,DEN
C
C CREATE A NEW FILE, TRIAL1.DAT, TO STORE THE DATA.
C
      OPEN(32,FILE='TRIAL1.DAT',STATUS='NEW')
C
C   R - SCALED SLANT RANGE IN KILOFEET
C   X - SCALED GROUND RANGE IN KILOFEET
C   Y - SCALED BURST HEIGHT IN KILOFEET
C  GR - ACTUAL RANGE IN FEET FROM GROUND ZERO
C   H - ACTUAL HEIGHT OF BURST IN FEET
C   M - SCALE FACTOR
C  PS - OVERPRESSURE IN PSI
C
C THESE TWO DO LOOPS GENERATE PEAK OVERPRESSURE VALUES FOR
C GROUND RANGES FROM 25 TO 350 FEET IN 25 FOOT INCREMENTS
C AND BURST HEIGHTS FROM 0 TO 400 FEET IN 25 FOOT
C INCREMENTS.  THEY ARE EXAMPLES ONLY.
C
      DO 30 GR=25,350,25
      DO 10 H=0,400,25
C
C SCALE RANGE AND HEIGHT TO 1 KT.
C
      M=1.
      X=(GR/M)/1000.
      Y=(H/M)/1000.
      R=SQRT(X**2+Y**2)
      Z=Y/X
C
C CALCULATE EACH PART OF THE BRODE EXPRESSION.
C
      AZ=1.22-((3.908*Z**2)/(1+810.2*Z**5))
C
      BZ1=(6.195*Z**18)/(1+1.113*Z**18)
      BZ2=(0.03831*Z**17)/(1+0.02415*Z**17)
      BZ3=0.6692/(1+4164*Z**8)
      BZ=2.321+BZ1-BZ2+BZ3
C
      CZ1=(1.149*Z**18)/(1+1.641*Z**18)
```

```
      CZ2=1.1/(1+2.771*Z**2.5)
      CZ=4.153-CZ1-CZ2
C
      DZ1=(25.76*Z**1.75)/(1+1.382*Z**18)
      DZ2=(8.257*Z)/(1+3.219*Z)
      DZ=-4.166+DZ1+DZ2
C
      EZ=1-((0.004642*Z**18)/(1+0.003886*Z**18))
C
      FZ1=(2.879*Z**9.25)/(1+2.359*Z**14.5)
      FZ2=(17.15*Z**2)/(1+71.66*Z**3)
      FZ=0.6096+FZ1-FZ2
C
      GZ=(1.83+5.361*Z**2)/(1+0.3139*Z**6)
C
      HZ1=(8.808*Z**1.5)/(1+154.5*Z**3.5)
      HZ2=(0.2905+64.67*Z**5)/(1+441.5*Z**5)
      HZ3=(1.389*Z)/(1+49.03*Z**5)
      DEN=(781.2-(123.4*R)+(37.98*R**1.5)+R**2)*(1+2*Y)
      HRY=(1.094*R**2)/DEN
      HZRY=HZ1-HZ2-HZ3+HRY
C
      MY1=(0.000629*Y**4)/(3.493E-09+Y**4)
      MY2=(2.67*Y**2)/(1+1E+07*Y**4.3)
      MY=MY1-MY2
C
      NY=5.18+(0.2803*Y**3.5)/(3.788E-06+Y**4)
C
      PS1=10.47/(R**AZ)
      PS2=BZ/(R**CZ)
      PS3=(DZ*EZ)/(1+FZ*R**GZ)
      PS4=MY/(R**NY)
      PS=PS1+PS2+PS3+HZRY+PS4
C
C WRITE THE DATA TO FILE 32 USING FORMAT 100 STATEMENT.
C
      WRITE(32,100) X,Y,PS
 100  FORMAT(F6.3,F8.3,F9.1)
C
  10  CONTINUE
  30  CONTINUE
      STOP
      END
```

This SAS program reads a data set, computes the variables, performs the regression analysis, and plots four graphs of residuals. Consult Freund and Littell (10) and the SAS User's Guides (21) and (22) for more detailed information.

```
/* THIS LINE SPECIFIES PRINTOUT SIZE. */
OPTIONS LINESIZE=75;

/* THESE LINES LOOK FOR THE DATA IN FILE TRIAL7.DAT, INPUT
THREE VARIABLES PER LINE, AND COMPUTE THE REMAINING
VARIABLES. */
DATA;
/*OVERPRESSURE RANGE 10 TO 1 PSI*/
  INFILE TRIAL7;
  INPUT X Y OVP @@;

/* CONVERT OVERPRESSURE FROM PSI TO KSI. */
  OVP=OVP/1000;          /* P */

/* THESE ARE ALL POSSIBLE COMBINATIONS OF VARIABLES FOR A
FIFTH DEGREE POLYNOMIAL EQUATION. */
  PSQ=OVP*OVP;           /* P2 */
  PCUB=OVP*OVP*OVP;      /* P3 */
  PQT=PSQ*PSQ;           /* P4 */
  PY=OVP*Y;              /* PY */
  P2Y=PSQ*Y;             /* P2Y */
  P3Y=PCUB*Y;            /* P3Y */
  P4Y=PQT*Y;             /* P4Y */
  YSQ=Y*Y;               /* Y2 */
  YCUB=Y*Y*Y;            /* Y3 */
  YQT=YSQ*YSQ;           /* Y4 */
  PYSQ=PSQ*YSQ;          /* P2Y2 */
  PYC=PCUB*YCUB;         /* P3Y3 */
  Y2P=YSQ*OVP;           /* Y2P */
  Y3P=YCUB*OVP;          /* Y3P */
  Y4P=YQT*OVP;           /* Y4P */
  P3Y2=PCUB*YSQ;         /* P3Y2 */
  P4Y2=PQT*YSQ;          /* P4Y2 */
  P4Y3=PQT*YCUB;         /* P4Y3 */
  PYQT=PQT*YQT;          /* P4Y4 */
  Y3P2=YCUB*PSQ;         /* Y3P2 */
  Y4P2=YQT*PSQ;          /* Y4P2 */
  Y4P3=YQT*PCUB;         /* Y4P3 */
  P5=PQT*OVP;            /* P5 */
  P5Y=P5*Y;              /* P5Y */
  P5Y2=P5*YSQ;           /* P5Y2 */
  P5Y3=P5*YCUB;          /* P5Y3 */
  P5Y4=P5*YQT;           /* P5Y4 */
  Y5=YQT*Y;              /* Y5 */
  Y5P=Y5*OVP;            /* Y5P */
  Y5P2=Y5*PSQ;           /* Y5P2 */
```

```
  Y5P3=Y5*PCUB;            /* Y5P3  */
  Y5P4=Y5*PQT;             /* Y5P4  */
  P5Y5=P5*Y5;              /* P5Y5  */

/* THIS IS THE REGRESSION PROCEDURE.  THE R AFTER THE MODEL
STATEMENT COMPUTES A VARIETY OF STATISTICS. */
PROC REG;
  MODEL X=OVP Y PY PSQ P2Y YSQ Y2P PYSQ PCUB P3Y YCUB
          Y3P PYC PQT P4Y P3Y2 P4Y2 P4Y3 YQT Y4P Y3P2
          Y4P2 Y4P3 PYQT P5 P5Y P5Y2 P5Y3 P5Y4 Y5 Y5P
          Y5P2 Y5P3 Y5P4 P5Y5 / R;

/* THE OUTPUT IS PLACED IN A NEW SAS DATA SET LABELED A.
THE RESIDUALS AND PREDICTED VALUES ARE ALSO DEFINED. */
  OUTPUT OUT=A P=PX R=RX RSTUDENT=RSTUDENT;
  ID OVP;

/* THESE COMMANDS PLOT FOUR GRAPHS OF RESIDUALS. */
PROC PLOT DATA=A;
  PLOT (RX RSTUDENT)*PX RX*Y RX*OVP / VREF=0;
/* THIS COMMAND EXECUTES THE PROGRAM. */
RUN;
```

This small SAS program reads a data set, establishes a data grid, builds a response surface, and plots a contour graph. Consult the SAS/GRAPH User's Guide (23) for more detailed information.

```
/* THIS LINE SPECIFIES PRINTOUT SIZE */
OPTION LINESIZE=75;
/* THIS LINE SPECIFIES GRAPHICS DEVICE */
GOPTIONS CBACK=BLACK DEVICE=TEK4107;
/* RENAME DATA FILE TRIAL7.DAT TO EXPER */
FILENAME EXPER 'TRIAL7.DAT';

/* THESE LINES TELL SAS THERE IS DATA; IT IS LOCATED IN FILE
EXPER; WHAT VARIABLES ARE BEING READ; AND TO EXECUTE READING
THE DATA */
DATA NEW;
INFILE EXPER;
INPUT X Y OVP;
RUN;

/* THIS PART ARRANGES THE DATA INTO A GRID */
PROC G3GRID DATA=NEW OUT=GRIDNEW;
TITLE 'RANGE FUNCTION';
GRID Y*OVP=X/ AXIS1=0 TO 2.4 BY .1 AXIS2=1 TO 10 BY .5;
RUN;
/* THIS PART PLOTS A RESPONSE SURFACE */
PROC G3D DATA = GRIDNEW;
PLOT Y*OVP=X /SIDE TILT=35 ROTATE=70;
RUN;

/* THIS PART PLOTS A CONTOUR GRAPH */
PROC GCONTOUR DATA=GRIDNEW;
PLOT Y*OVP=X / LEVELS=0.5 TO 6.5 BY .5;
RUN;
```

```
      PROGRAM RANGE
      IMPLICIT NONE
C THIS PROGRAM COMPUTES AND PRINTS OUT THE GROUND RANGE IN
C FEET FROM GROUND ZERO AS A FUNCTION OF WEAPON YIELD,
C SURFACE PEAK OVERPRESSURE IN PSI, AND BURST HEIGHT IN
C FEET.  IT WAS BUILT BY MAJ WOLCZEK, GST88M, AFIT, AND
C APPROXIMATES AN INVERSE TO THE BRODE EXPRESSION ON PAGE 60
C OF PSR REPORT 1419-3.
C
      REAL X,Y,P,PSQ,PCUB,PQT,P5,PY,P2Y,P3Y,P4Y,P5Y,YSQ,YCUB
      REAL YQT,Y5,PYSQ,PYC,PYQT,P5Y5,Y2P,Y3P,Y4P,Y5P,P3Y2
      REAL P4Y2,P5Y2,P4Y3,P5Y3,P5Y4,Y3P2,Y4P2,Y5P2,Y4P3,Y5P3
      REAL Y5P4,GR,H,PS,M,W
      INTEGER N
C
C     X - SCALED GROUND RANGE IN KILOFEET
C     Y - SCALED BURST HEIGHT IN KILOFEET
C     P - PEAK OVERPRESSURE IN KSI
C    GR - ACTUAL GROUND RANGE IN FEET
C     H - ACTUAL HEIGHT OF BURST IN FEET
C    PS - PEAK OVERPRESSURE IN PSI
C     M - SCALE FACTOR
C     W - ACTUAL WEAPON YIELD IN KILOTONS
C
   20 PRINT*, 'PLEASE INPUT WEAPON YIELD IN KILOTONS.'
      READ*, W
      PRINT*, 'PLEASE INPUT PEAK OVERPRESSURE IN PSI.'
      READ*, PS
      PRINT*, 'PLEASE INPUT HEIGHT OF BURST IN FEET.'
      READ*, H
C
C SCALE HEIGHT TO 1 KT AND CONVERT TO KFT.
C
      M=W**.333333
      Y=(H/M)/1000.
C
C CONVERT PEAK OVERPRESSURE TO KSI.
C
      P=PS/1000.
C
C THESE ARE THE VARIABLES OF THE POLYNOMIALS.
C
      PSQ=P*P
      PCUB=PSQ*P
      PQT=PSQ*PSQ
      P5=PQT*P
      PY=P*Y
      P2Y=PSQ*Y
      P3Y=PCUB*Y
```

```
      P4Y=PQT*Y
      P5Y=P5*Y
      YSQ=Y*Y
      YCUB=YSQ*Y
      YQT=YSQ*YSQ
      Y5=YQT*Y
      PYSQ=PSQ*YSQ
      PYC=PCUB*YCUB
      PYQT=PQT*YQT
      P5Y5=P5*Y5
      Y2P=YSQ*P
      Y3P=YCUB*P
      Y4P=YQT*P
      Y5P=Y5*P
      P3Y2=PCUB*YSQ
      P4Y2=PQT*YSQ
      P5Y2=P5*YSQ
      P4Y3=PQT*YCUB
      P5Y3=P5*YCUB
      P5Y4=P5*YQT
      Y3P2=YCUB*PSQ
      Y4P2=YQT*PSQ
      Y5P2=Y5*PSQ
      Y4P3=YQT*PCUB
      Y5P3=Y5*PCUB
      Y5P4=Y5*PQT
C
C THIS PORTION OF THE PROGRAM USES THE CURVE FITS FOR FIVE
C REGIONS OF PEAK OVERPRESSURE (1-10, 10-100, 100-1000,
C 1000-10000, AND 10000-100000 PSI) TO COMPUTE SCALED GROUND
C RANGE IN KFT.
C
      IF (P.GT.100.) THEN
      PRINT*, 'THE HIGH LIMIT FOR PEAK OVERPRESSURE IS'
      PRINT*, '100,000 PSI.'
      GO TO 50
      ELSE IF (P.GT.10.) THEN
C
C THE OVERPRESSURE RANGE COVERED IS 10000 TO 100000 PSI.
C
      X=.09125435-.00259593*P-.011649*PY+.0000538051*PSQ
     #  +.0001111895*P2Y+1.55066730*Y2P-.0156667*PYSQ
     #  -5.2376E-07*PCUB+.00004916175*P3Y2+97.31778176*YCUB
     #  -25.706*Y3P+1.86726E-09*PQT-776.975*YQT
     #  +80.37406298*Y4P+.10802327*Y3P2+.01220085*Y4P3
     #  -.000106692*PYQT
C
      ELSE IF (P.GT.1.) THEN
C
```

```
C THE OVERPRESSURE RANGE COVERED IS 1000 TO 10000 PSI.
C
      X=.20154762-.0621137*P+.26548571*Y+.01362242*PSQ
     #  -7.29235*YSQ+.07036665*PYSQ-.00141394*PCUB
     #  +39.26591463*YCUB+47.69269458*Y3P+1.41551464*PYC
     #  +.00005402714*PQT-.0671128*P4Y3-314.529*Y4P
     #  -12.1885*Y3P2+64.01100943*Y4P2-8.28738*Y4P3
     #  +.42330598*PYQT
C
      ELSE IF (P.GT.0.1) THEN
C
C THE OVERPRESSURE RANGE COVERED IS 100 TO 1000 PSI.
C
      X=.49576495-1.76963*P+.2873705*Y+4.11651645*PSQ
     #  -7.91446*YSQ+30.88152847*Y2P-157.536*PYSQ
     #  -4.38405*PCUB+237.78055884*P3Y2+30.88701918*YCUB
     #  -90.0484*Y3P-1383.45*PYC+1.69350254*PQT
     #  -109.732*P4Y2+631.54888112*P4Y3-29.9186*YQT
     #  +876.59060473*Y3P2-1444.15*Y4P2+2183.11803*Y4P3
     #  -935.61*PYQT
C
      ELSE IF (P.GT.0.01) THEN
C
C THE OVERPRESSURE RANGE COVERED IS 10 TO 100 PSI.
C THIS IS A FIFTH DEGREE POLYNOMIAL.
C
      X=1.62275752-83.2005*P+2705.11147*PSQ+6.12568509*YSQ
     #  -546.803*Y2P+10261.68969*PYSQ-45666.2*PCUB
     #  -56907.7*P3Y2-14.3825*YCUB+1654.94706*Y3P
     #  +375478.28531*PQT+15.10099190*YQT-1985.89*Y4P
     #  -22491*Y3P2+18051.29615*Y4P2+4798899.81*PYQT
     #  -1186924*P5+8777484.91*P5Y3-48910776*P5Y4
     #  -5.23814*Y5+660.83710834*Y5P-5735942*Y5P4
     #  +49158205.9*P5Y5
C
      ELSE IF (P.GE.0.001) THEN
C
C THE OVERPRESSURE RANGE COVERED IS 1 TO 10 PSI.
C THIS IS A FIFTH DEGREE POLYNOMIAL.
C
      X=8.23926119-5306.71*P+6.05714948*Y-5471.77*PY
     #  +1828923.98*PSQ+1686307.36*P2Y-2.89962*YSQ
     #  +4167.17167*Y2P-352120*PYSQ-31.9297925E+7*PCUB
     #  -27.9705884E+7*P3Y+.87118870*YCUB-1187.34*Y3P
     #  +162.39402197E+6*PYC+269.81456091E+8*PQT
     #  +240.17907175E+8*P4Y-103.60283247E+8*P4Y3
     #  -219.221*Y4P-855214*Y3P2+501833.53514*Y4P2
     #  -47.312304E+6*Y4P3-8.75407E+11*P5-8.08007E+11*P5Y
     #  +3361.23611805E+8*P5Y3-.0104459*Y5+34.21894224*Y5P
     #  -31509.7*Y5P2-11250907*Y5P3+203.1223752E+7*Y5P4
     #  -6059.3995968E+7*P5Y5
C
```

```
      ELSE
      PRINT*, 'THE LOW LIMIT FOR PEAK OVERPRESSURE IS 1'
      PRINT*, 'PSI.'
      GO TO 50
      END IF
C
C THE ABOVE POLYNOMIALS PRODUCE NEGATIVE VALUES OF X IN
C THOSE REGIONS BEYOND X=0.  SINCE SUCH A PHENOMENON IS
C IMPOSSIBLE, THESE VALUES ARE SET TO ZERO.
C
      IF (X.LT.0) THEN
      X=0
      END IF
C
C SCALE GROUND RANGE BACK TO ACTUAL WEAPON YIELD AND CONVERT
C TO FEET.
C
      GR=(X*1000.)*M
C
      PRINT 100, W
      PRINT 110, PS
      PRINT 120, H
      PRINT 130, GR
 100  FORMAT(' FOR A',F12.2,' KT WEAPON WITH:')
 110  FORMAT('     PEAK OVERPRESSURE =',F12.2,' PSI,')
 120  FORMAT('     BURST HEIGHT =',F12.2,' FT,')
 130  FORMAT(' GROUND RANGE =',F12.2,' FT.')
  50  PRINT*
      PRINT*, 'IF YOU WISH TO REPEAT THE CALCULATIONS,'
      PRINT*, 'ENTER A 1; IF NOT, PRESS ANY OTHER NUMBER.'
      READ*, N
      IF (N.EQ.1) GO TO 20
      STOP
      END
```

Appendix C: Contour Graphs

This section contains comparisons of predicted to actual data for Regions 1 through 5.

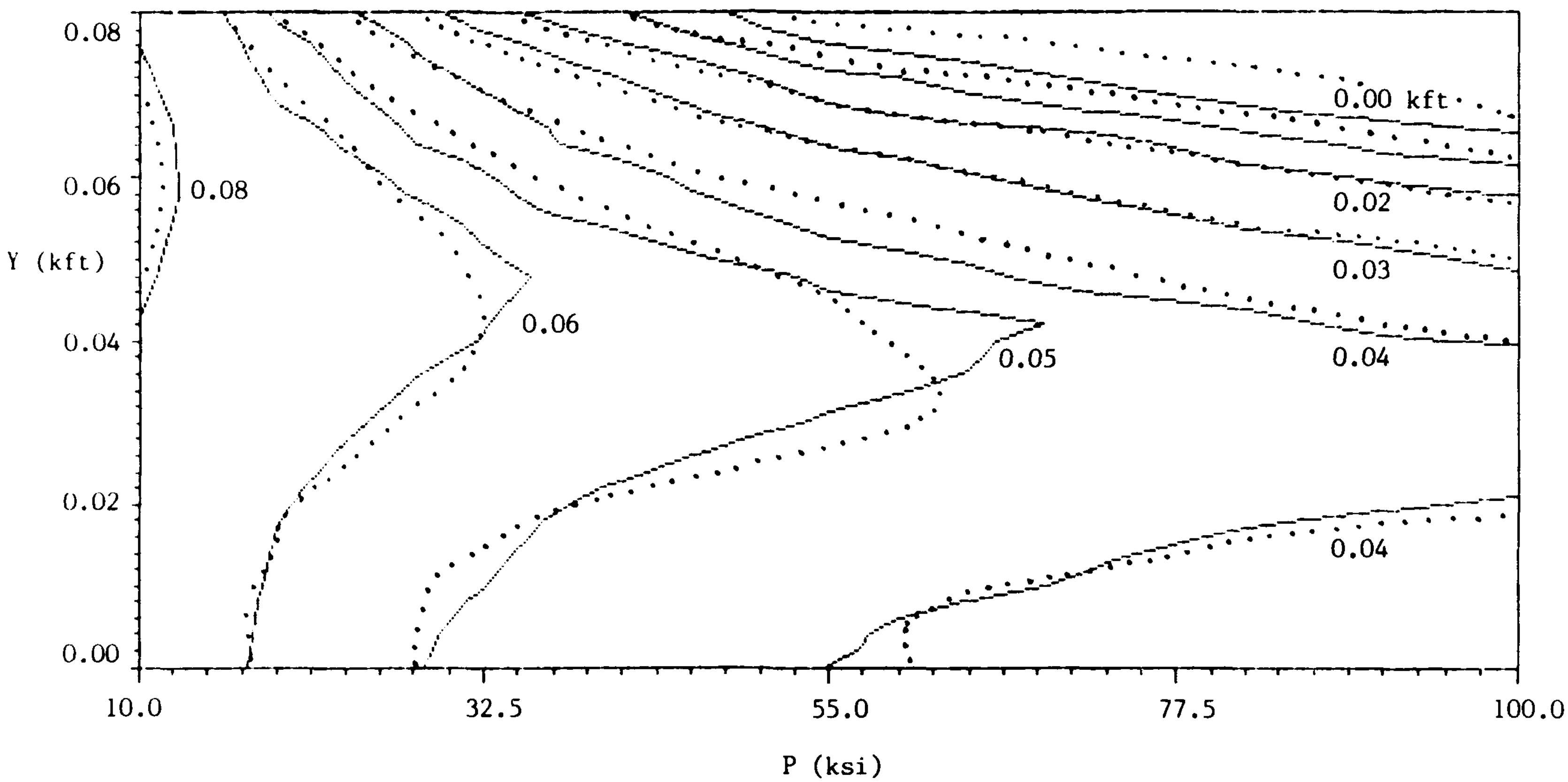

Figure 27. Comparison of predicted (dotted) versus actual constant scaled ground range contours for Region 1 (10,000-100,000 psi) plotted as a function of peak overpressure and scaled height of burst.

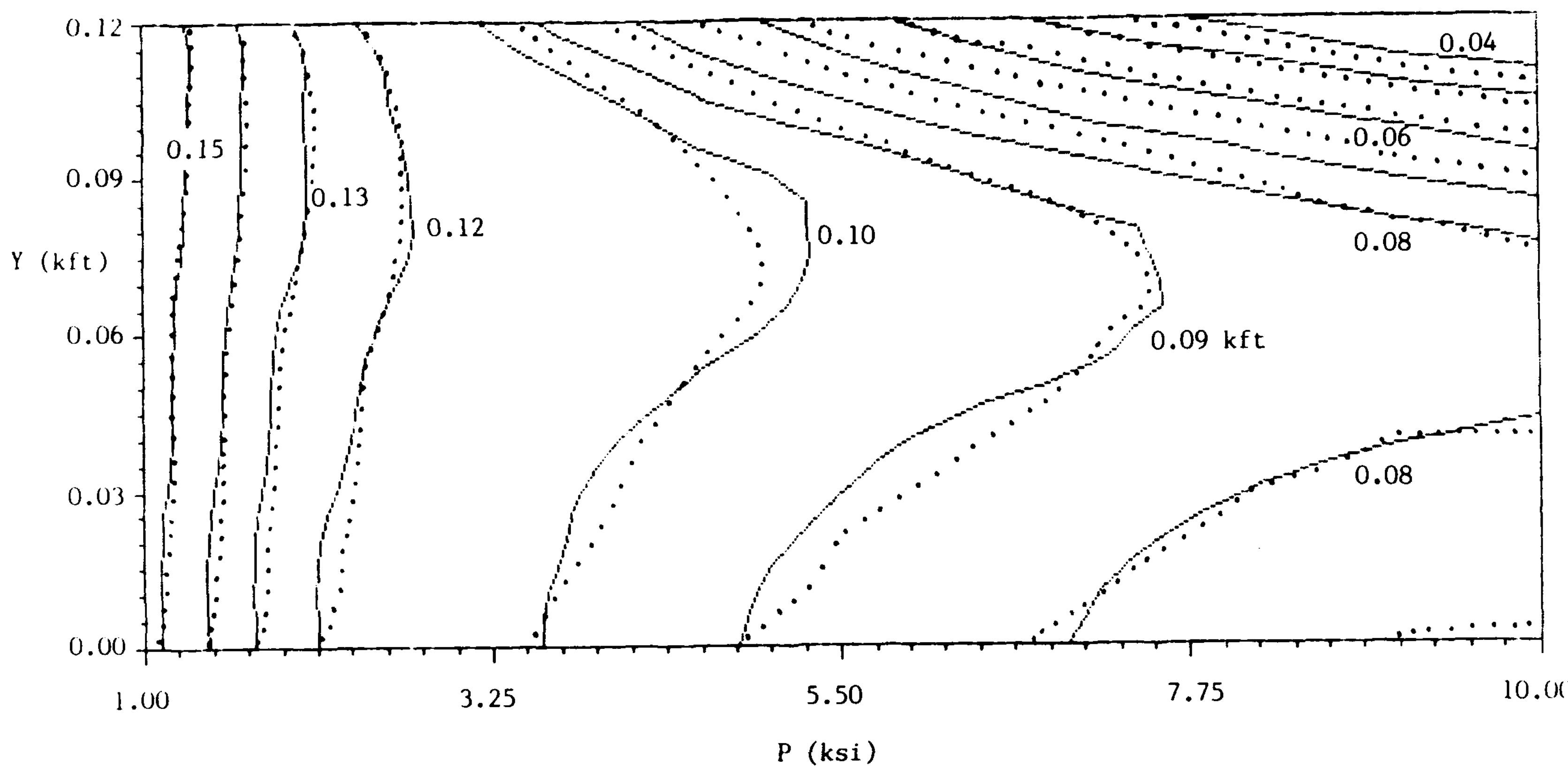

Figure 28. Comparison of predicted (dotted) versus actual constant scaled ground range contours for Region 2 (1,000-10,000 psi) plotted as a function of peak overpressure and scaled height of burst.

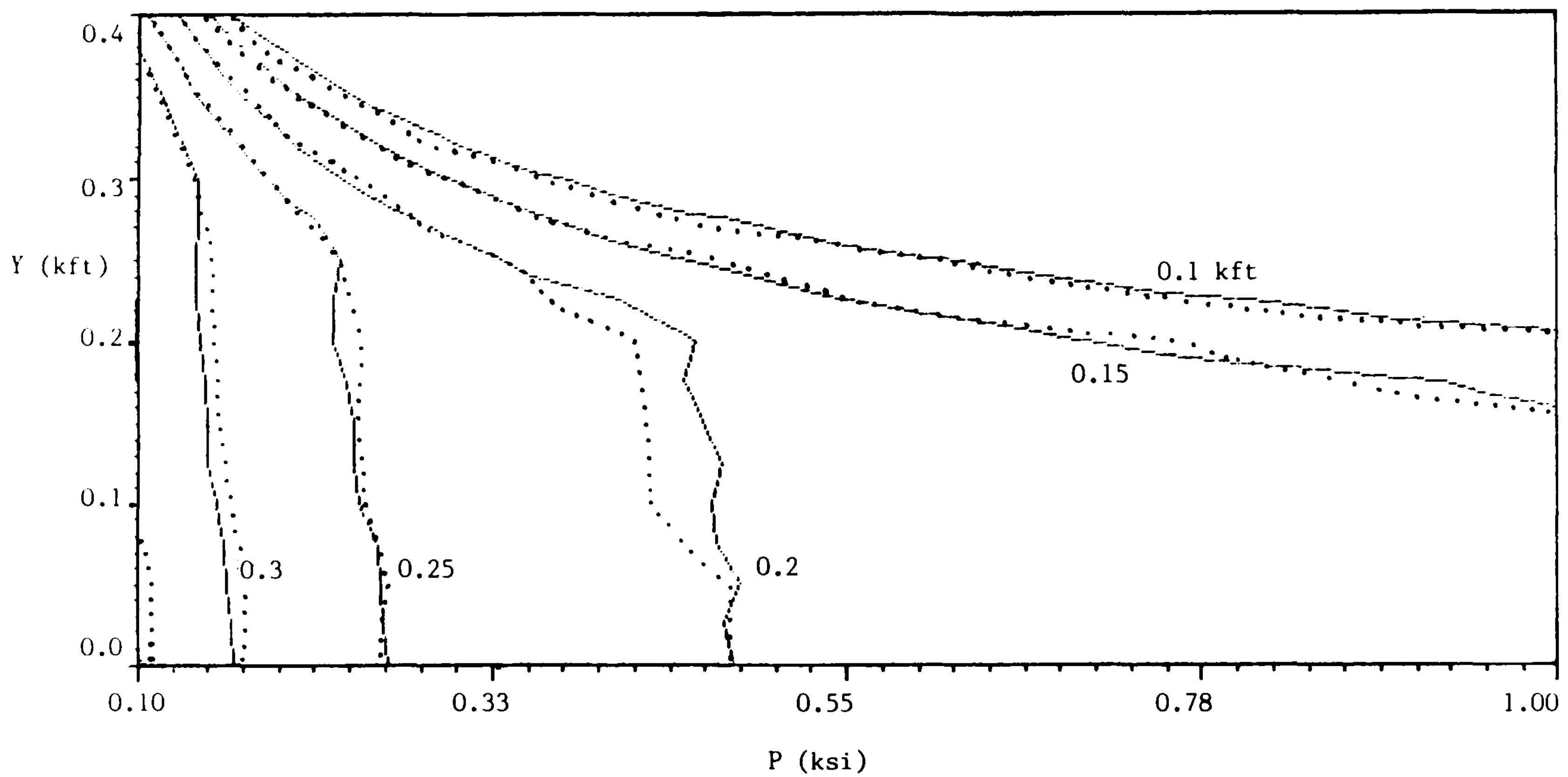

Figure 29. Comparison of predicted (dotted) versus actual constant scaled ground range contours for Region 3 (100-1,000 psi) plotted as a function of peak overpressure and scaled height of burst.

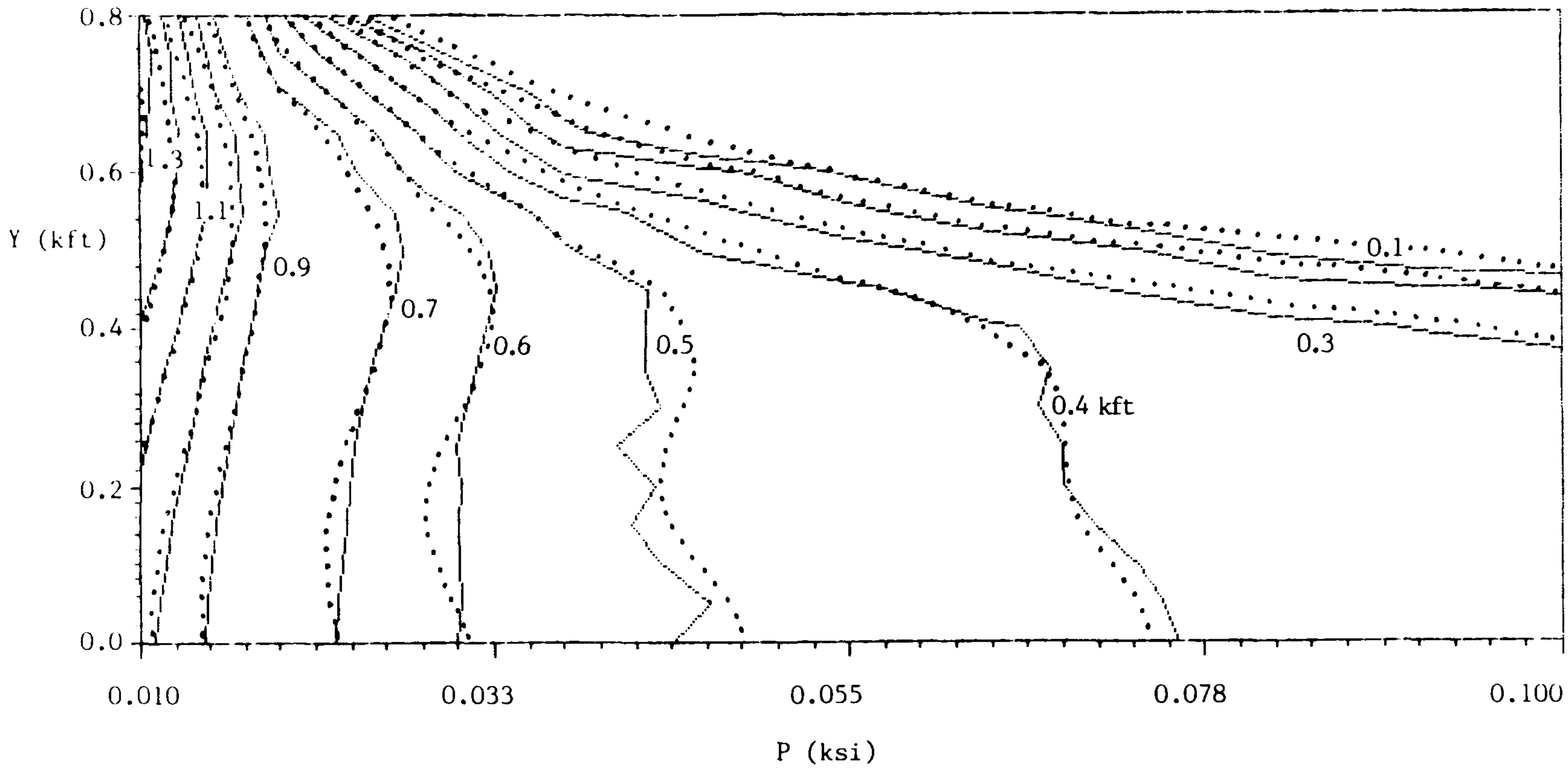

Figure 30. Comparison of predicted (dotted) versus actual constant scaled ground range contours for Region 4 (10-100 psi) plotted as a function of peak overpressure and scaled height of burst.

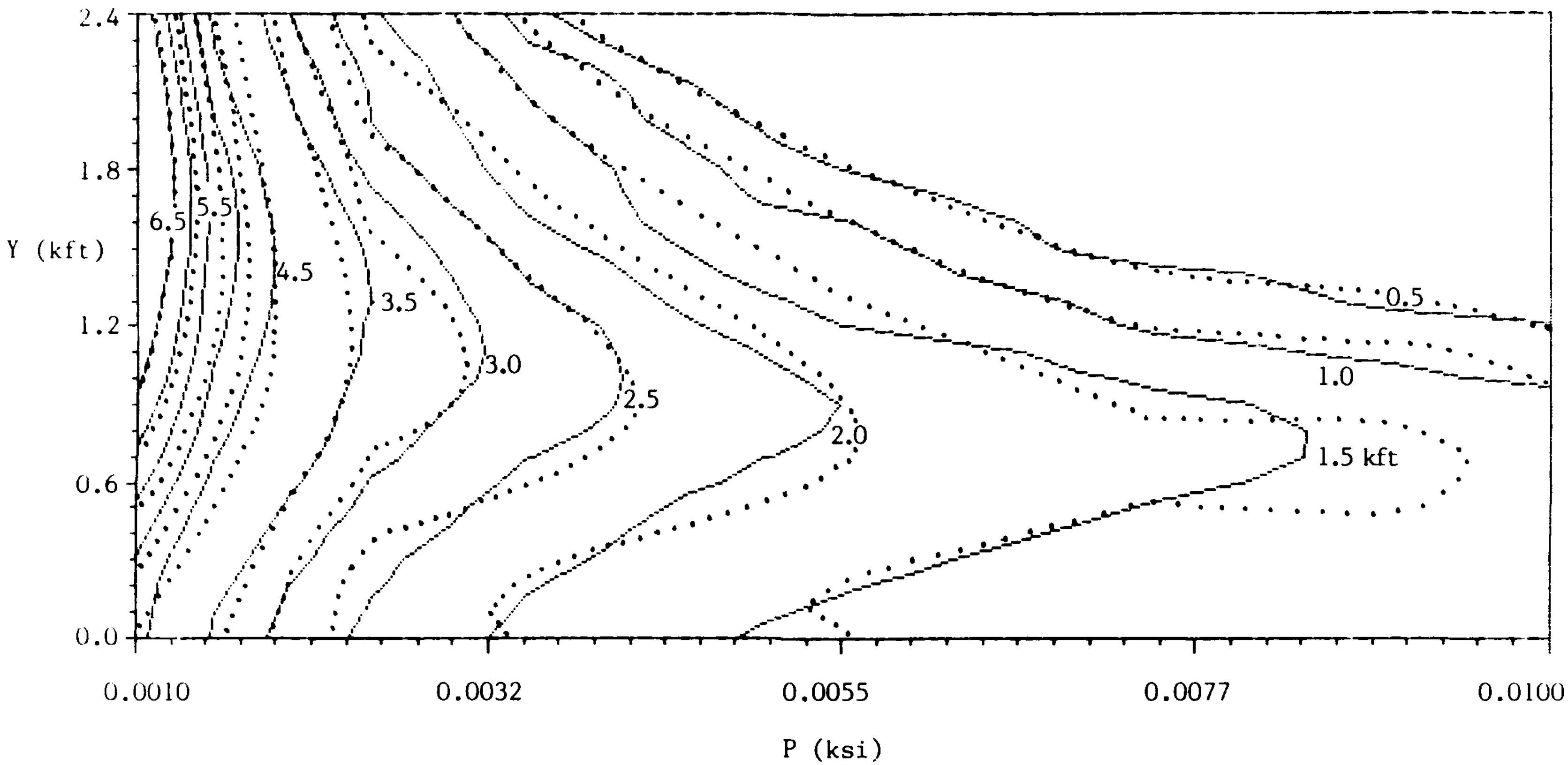

Figure 31. Comparison of predicted (dotted) versus actual constant scaled ground range contours for Region 5 (1-10 psi) plotted as a function of peak overpressure and scaled height of burst.

Appendix D: Additional Methodology

This appendix provides more information on the methodology used to fit ground range as a function of single independent variables. This methodology, outlined in Chapter 2, was basically a two step process. The first step involved selecting an equation form to find scaled ground range (X) as a function of peak overpressure (P) for a series of constant values of scaled burst height (Y). For each Y, the approximation generated a unique set of coefficients (A, B, etc.). Then, in the second step, analytic expressions for these coefficients were determined as a function of Y. The process seemed simple enough but in actuality had a number of problems associated with it. These problems were considered sufficiently difficult to reject this methodology from further consideration.

The first step involved selecting an equation form that fit the desired range of data. As mentioned previously, Region 2 (1000 to 10,000 psi peak overpressure) was selected for the initial trials. Nine values of Y ranging from 0 to 200 feet in 25 foot intervals were selected, and nineteen values of X and corresponding values of P were generated as data for each Y interval. The data sets were plotted to view the nature of the curves for equation selection. Some selected examples of these curves are provided in Figure 32. Examination of Figure 32 shows that the curves range from

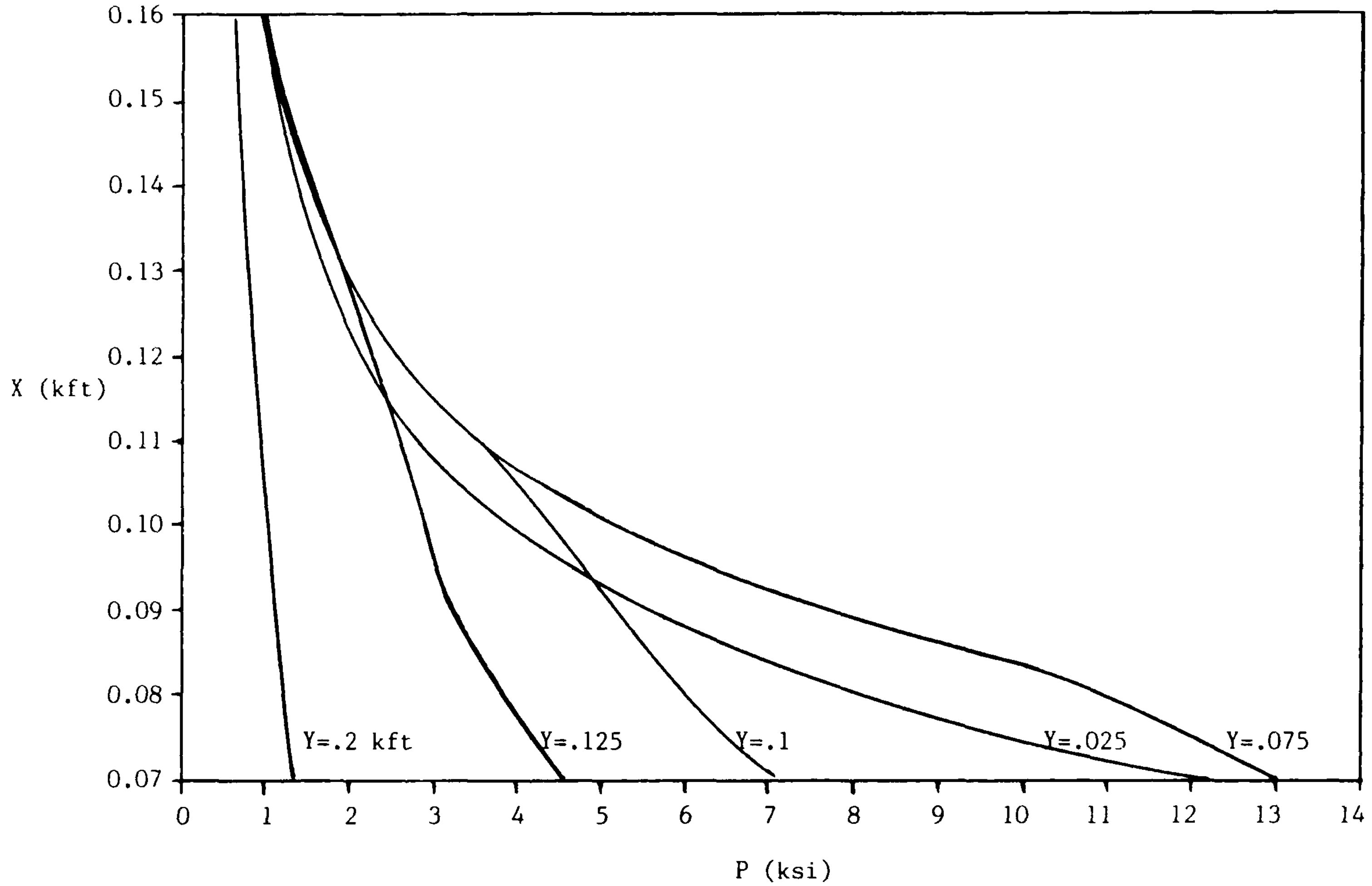

Figure 32. Scaled ground range (X) plotted as a function of peak overpressure (P) for selected values of scaled burst height (Y).

exponential in nature at Y = .025 kft to almost linear at Y = .2 kft. The objective was to find an equation that could fit the entire range of curves.

The following analytical expressions were tested against the curves in Figure 32.

$$X = AP^{B} \tag{11}$$

$$X = Ae^{BP} \tag{12}$$

$$X = A + BP^{C} \tag{13}$$

$$X = A + Be^{CP} \tag{14}$$

$$X = A + B(e^{CP})(P^{D}) \tag{15}$$

$$X = (A + Be^{CP})(P^{D}) \tag{16}$$

$$X = A + Be^{CP} + P^{D} \tag{17}$$

$$X = A + Be^{CP} + DP^{E} \tag{18}$$

$$X = \frac{A + BP^{C}}{(.0001 + P^{D})(.0001 + EP^{F})} \tag{19}$$

$$X = A + BP + CP^{2} + DP^{3} + EP^{4} \tag{20}$$

where

X = scaled ground range in kft

P = peak overpressure in ksi

A, B, C, D, E = various coefficients

e = natural logarithm

The SAS PROC NLIN procedure was used to fit these nonlinear regression models by least squares (22:576). Equations (15) and (16) failed to converge in most cases and

were rejected. Equations (11), (12), (13), (14), (18), and (19) provided good fits for data defined by low and high values of Y. However, these equations provided poor fits for the data defined by Y = .075 to .125. Equation (17) fit the data defined by low and middle values of Y but failed to adequately fit the data defined by high values of Y. The fourth degree polynomial equation, (20), provided the best overall fit for the entire range of curves.

Table 6 shows the coefficients of the best fits for the data using Equation (20).

Table 6. Optimal output coefficients for scaled ground range as a function of peak overpressure using Equation (20).

Y	A	B	C	D	E	SS ERROR
0	.2052	-.065	.01448	-.0015	.000057	.000027
25	.1986	-.0559	.01116	-.001018	.000034	.000045
50	.1863	-.0390	.00572	-.000364	.000008	.00013
75	.1904	-.0422	.00737	-.000591	.000017	.000037
100	.2151	-.0766	.02355	-.003616	.000199	.000003
125	.2004	-.0551	.01616	-.004518	.000495	.000016
150	.0752	.2910	-.3056	.1109	-.01388	.000009
175	.4182	-.4863	.25806	-.05051	.0004	.000013
200	.8966	-2.6374	3.5006	-2.137	.48567	.000003

Since the second step of the fitting procedure involved finding analytical expressions for the coefficients as a function of Y, some predictable behavior in the coefficients was desired. The coefficients in Table 6 were too random to obtain an adequate fit. Therefore, the optimal fits were manipulated to reduce the randomness of the coefficients.

However, these manipulations of the coefficients also increased the error in the fit. Table 7 depicts the coefficients after a series of manipulations of the original fits to make the data conform to some predictable pattern.

Table 7. Revised output coefficients for scaled ground range as a function of peak overpressure using Equation (20).

Y	A	B	C	D	E	SS ERROR
0	.20523	-.065	.01448	-.0015	.0000571	.000027
25	.19855	-.05594	.01116	-.001018	.0000335	.000045
50	.18639	-.03900	.005723	-.000364	.0000081	.00013
75	.19036	-.04224	.007374	-.000591	.0000169	.000039
100	.197	-.05163	.01259	-.001716	.0000862	.000024
125	.21082	-.06479	.016	-.003	.0002424	.000047
150	.25554	-.1209	.02857	-.0039	.0003	.000052
175	.30067	-.21526	.0592	-.005	.0003	.000058
200	.2941	-.2587	.07827	-.0065	.00025	.000046

This table of data was then used to find analytic expressions for the coefficients as a function of Y.

Each set of coefficients was plotted against Y to discover the nature of the curves. These plots are displayed in Figures 33 through 35. Since they resemble curves characteristic of polynomials, it was decided to use polynomial equations to fit the data.

During the manipulation process, it was discovered that the polynomial equations were very sensitive to round-off errors. Small changes in the optimal coefficients produced unexpected changes in the shape of a curve. The coefficients of the fits for Y = 0 through 75 feet were

especially sensitive to changes because their error was higher than the rest. Consequently, any analytical approximation with a coefficient as the dependent variable had to fit those sensitive coefficients almost exactly. However, it was impossible to accomplish such an exacting fit, even with further manipulation of the data.

Polynomial models up to seventh degree were examined. The fits were very good for normal data, however, they were not sufficiently exact to prevent significant and unexpected errors. Attempting to manually manipulate the data was both time consuming and complex. It was concluded that since polynomials produced the best results, a computerized method was required to do what had previously been accomplished manually. Therefore, a computerized procedure was sought that could fit a polynomial model with two independent variables. This procedure, in the form of SAS PROC REG, is covered in the main body of this thesis.

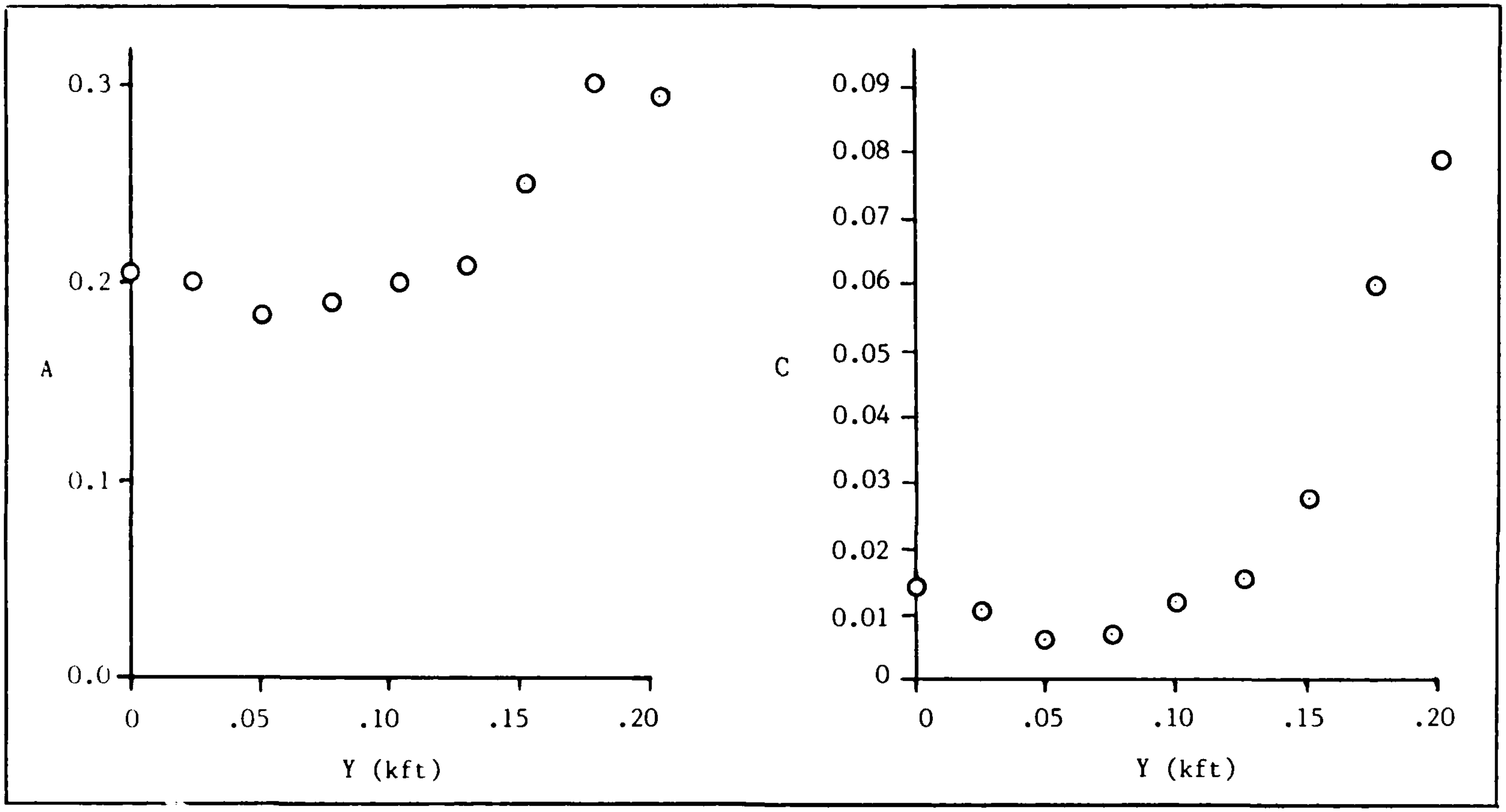

Figure 33. Coefficients A and C plotted as a function of burst height (Y).

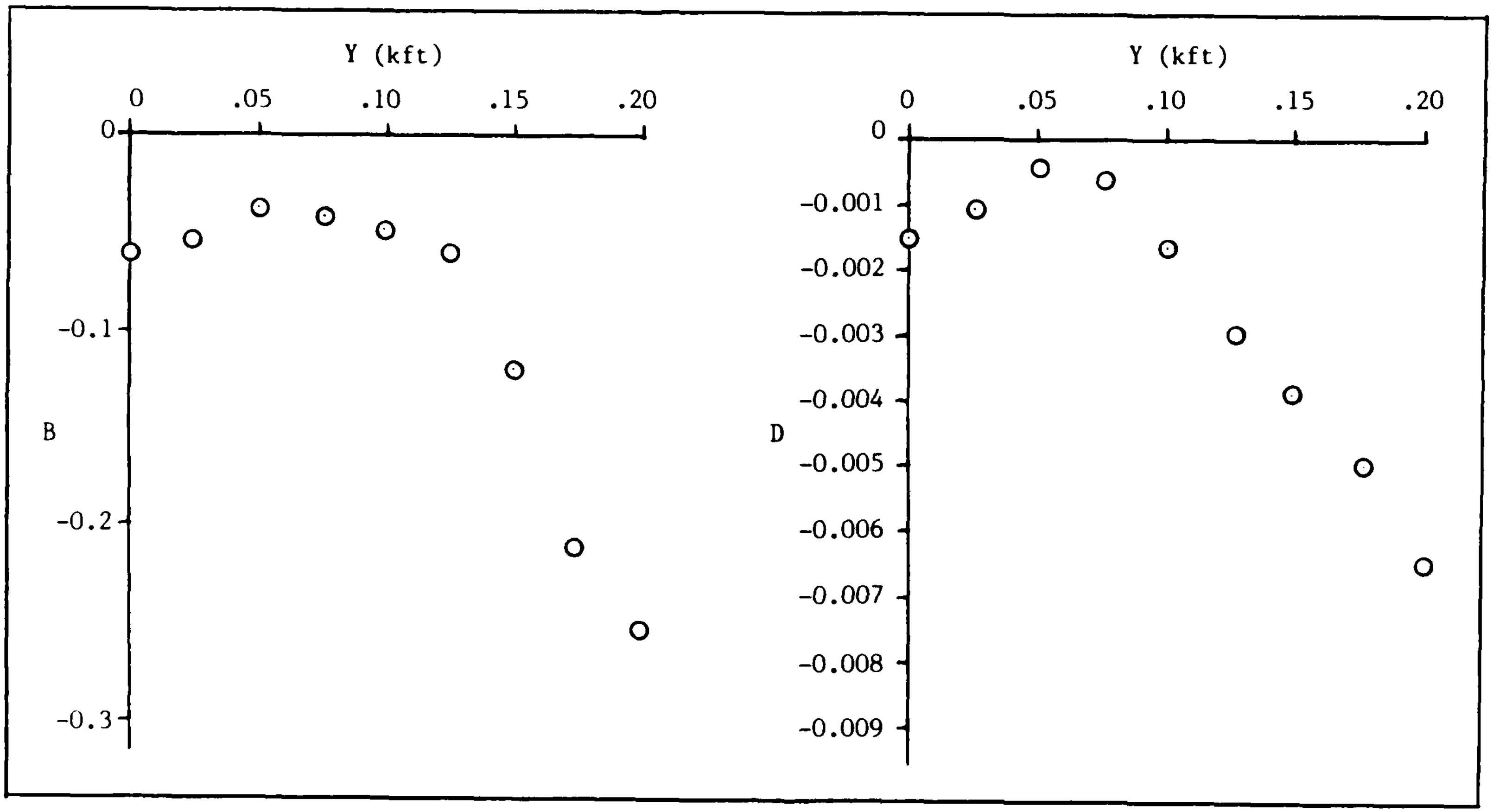

Figure 34. Coefficients B and D plotted as a function of burst height (Y).

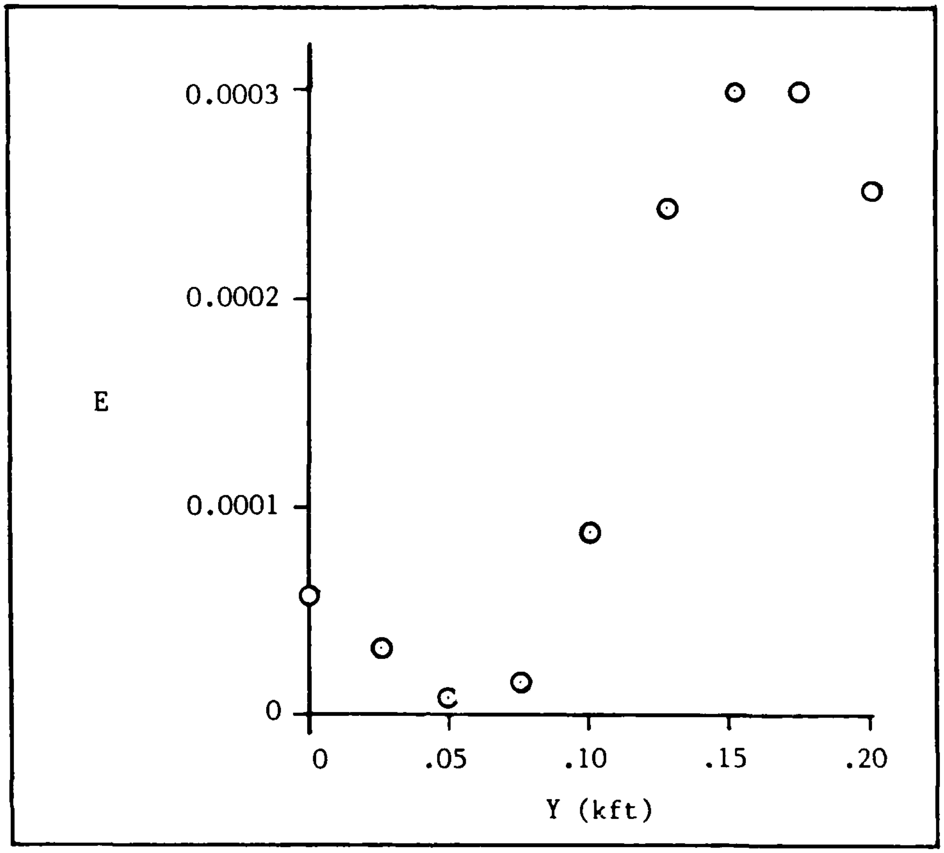

Figure 35. Coefficient E plotted as a function of burst height (Y).

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4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GST/ENP/88M-2	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION School of Engineering 6b. OFFICE SYMBOL (If applicable) AFIT/ENP	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Air Force Institute of Technology (AU) Wright-Patterson AFB, Ohio 45433-6583	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Air Force Center for Studies and Analysis 8b. OFFICE SYMBOL (If applicable) AFCSA/SASM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) Pentagon Washington, D.C. 20330-5420	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)

DIRECT DETERMINATION OF RANGE FROM CURRENT NUCLEAR OVERPRESSURE EQUATIONS

12. PERSONAL AUTHOR(S)
Roger S. Wolczek, B.S., Maj, USAF

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
MS Thesis	FROM ____ TO ____	1988 March	119

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
19	11		Nuclear Weapons, Weapons Effects; Overpressure;
12	01		Curve Fitting

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Thesis Advisors: Ronald F. Tuttle, Lt Col, USAF
Deputy Head and Assistant Professor of Nuclear Engineering

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Assistant Professor of Operations Research

Approved for public release: IAW AFR 190-1.
Lynn E. Wolaver 15 Jun 88
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The Brode expression determines peak overpressure as a function of scaled ground range from ground zero and scaled height of burst for nuclear explosions occurring at low altitudes. To calculate the scaled ground range as a function of peak overpressure and scaled height of burst presently requires an iterative numerical method to invert the Brode expression. This study developed analytical expressions to directly compute scaled ground range from ground zero as a function of peak overpressure and scaled height of burst for a nuclear explosion.

Since the Brode expression was an empirical fit of actual and predicted data, a curve fitting approach was selected over attempting to mathematically invert the expression. The Brode expression was used to both generate the data and evaluate the quality of any new expression. Acceptable error was specified as ten percent of the actual ground range for those regions of interest. The data range was sufficiently large to warrant breaking the problem up into five smaller segments. Each segment of the problem was solved by using least squares curve fitting on the SAS System.

Five analytical expressions in the form of polynomial equations were developed spanning peak overpressures from 1 to 100,000 psi. These polynomial equations were then combined into a Fortran 77 computer program which generated ground range directly from inputs of weapon yield, peak overpressure, and weapon burst height.

In most cases the error of the new approximation was well below ten percent of the actual ground range. There were two instances where the error was 10.9 and 11.5 percent of the ground range. These two cases were isolated and not indicative of the overall fit.